

# Norwegian upper secondary school students' performance in solving algebraic inequalities

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## Abstract

Inequalities arguably play an important role in mathematics, and student performance in this area may have a significant influence on their ability to learn algebra and calculus.

Furthermore, the topic is included in the intended curriculum of all the participating countries in TIMSS Advanced, as well as in the NCTM Standards. In this paper, the focus is on Norwegian upper secondary school students' mathematical performance when solving algebraic inequalities, and we relate this to curricular objectives and features of the problem presentation such as the wording of the task to be solved.

**Keywords: Mathematics, Algebraic Inequalities, TIMSS Advanced, Upper Secondary School**

## Introduction

While being a part of algebra, the topic of inequalities also pervades other areas like analysis and numerical methods. Inequalities arguably play an important role in mathematics, and the topic is included in the intended curriculum of all the participating countries in TIMSS Advanced, as well as in the NCTM Standards. Examining students' performance in solving algebraic inequalities is interesting, not only because it is an important mathematical topic, but also because studying students' performance on algebraic inequalities can provide an opportunity to explore several aspects that can reveal their understanding of algebra. This may include exploring their performance when manipulating symbolic expressions, their understanding of equivalence and how inequalities differ from equations, and their ability to use graphical arguments. It may also provide information regarding their ability to transform a mathematical statement from one type of representation to another (see figure 2).

There is a growing amount of literature on how inequalities are taught in schools, as well as on the difficulties students face when they solve inequalities. In some early texts, focusing on instructional approaches, specific solution strategies like using the sign-chart method (Dobbs & Peterson, 1991) or graphical methods (Dreyfus & Eisenberg, 1985) were recommended. Recently, researchers have concentrated more on students' understanding of inequalities. For instance, Tsamir and Almog (2001) investigated students' solution strategies and difficulties when faced with various types inequalities (linear, quadratic, rational, and square root). Their results showed that using graphical representations usually led to correct solutions, while difficulties with approaches based on algebraic manipulations arose when students failed to recognise the difference between inequalities and equations. One example of this is students' tendency to divide or multiply with non-positive factors (Tsamir & Almog, 2001). These results are supported by other studies, where authors point out that traditional instruction treat inequalities as a "relative" of equations, with a corresponding emphasis on algebraic solution techniques, and document the difficulties students face in the transition from equations to inequalities (Boero & Bazzini, 2004; Garuti, Bazzini, & Boero, 2001).

With respect to graphical solutions, Bazzini et al. (2001) have examined how different students reason when they face algebraic expressions. They find that students who have followed an instructional program where they are encouraged to work with different representations (algebraic expression, graphical representation) of the same mathematical object to a greater extent can work meaningful with algebra than students who have followed an instructional program that primarily focuses on the standard (algebraic) solution techniques. This is in line with the results of Tsamir and Almog (2001), who found that students were more successful when solving inequalities using a graphical approach. However, other researchers have pointed out that using graphical representations are demanding for students, and that this does not necessarily lead to errorless solutions (Sackur, 2004). Research have also suggests that students often have a limited understanding of the relationship between algebraic and graphical representations, and that they strongly prefer to solve tasks algebraically rather than utilising graphical information - even in cases where a graphical solution would be much easier (Knuth, 2000).

At the 28<sup>th</sup> PME conference, a working group was devoted to algebraic equations and inequalities (Bazzini & Tsamir, 2004)). The participating researchers addressed a variety of difficulties occurring in students' solutions of inequalities, and to some extent suggested

reasons for these difficulties. Initiatives like this shows that although researchers approach this field of study in different ways, a common goal is to investigate ways to promote performance on algebraic inequalities by analysing students' reactions to various representations of inequalities in different contexts, and to consider the way this topic was taught. This reflects the mathematics education community's considerable (and growing) interest in examining students' understanding of mathematical notions, their strategies and difficulties when working with various mathematical tasks, and how these strategy choices and difficulties might be explained.

In this study, the focus will be on the performance level of Norwegian upper secondary school students' in solving algebraic inequalities. We will relate this to curricular objectives and features of the problem presentation such as the wording of the task to be solved. Special attention will be given to the context within which the mathematical task is presented. Earlier research on students in comprehensive school has pointed out some critical aspects of an item for the achievement level of Norwegian students. This include features such as whether the task is presented as applied or pure mathematics, whether or not the task require formal mathematical knowledge or not, and whether or not the item invites to a straightforward solution using a calculator. Such factors seem to have a significant influence on the achievement of Norwegian students (Grønmo, 2010; Grønmo & Olsen, 2006; Grønmo, Onstad, & Pedersen, 2010).

## **Theoretical framework**

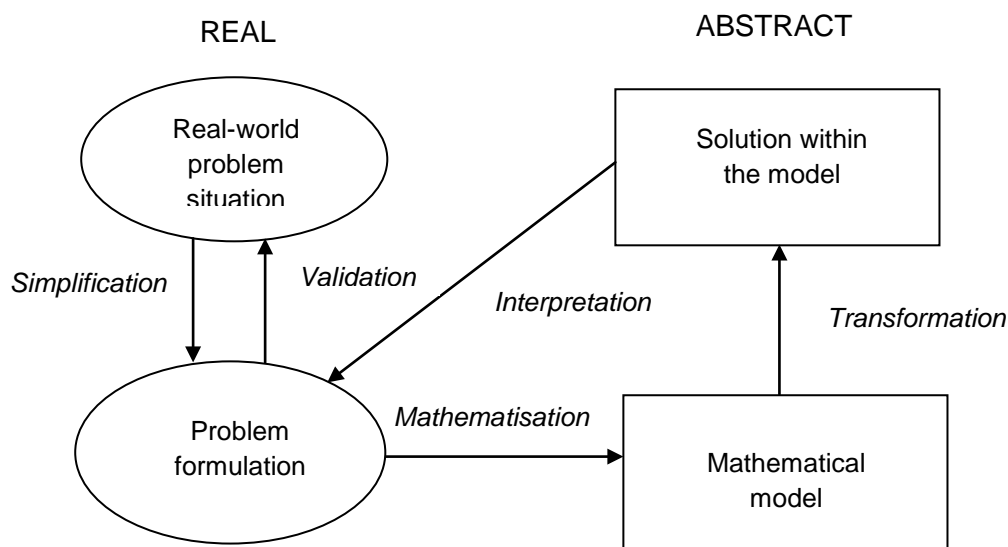
The theoretical framework of this paper combines two perspectives that both are related to how mathematical tasks are presented: the item context (applied or pure mathematics) and the choice of representation (symbolic or graphic).

### **Pure and applied mathematics**

In Norway, as in other Nordic countries, applied mathematics presented in a "real-world" context has been a driving force for school mathematics (Grønmo, 2010; Grønmo, Kjærnsli, & Lie, 2004). Figure 1 displays a commonly accepted model for the relation between pure and applied mathematics. Pure mathematics is on the right side of the figure, while the full circle of applied mathematics involves both sides of the figure. This illustrate that to apply mathematics, students need to have some knowledge in pure mathematics to find a correct solution. Applied mathematics can therefore be seen as more complex than pure mathematics,

if the same mathematics is involved in the two cases. Gardiner (2004) argues extensively that even if the ability to use mathematics to solve daily life problems is a main goal for school mathematics, this cannot be seen as an alternative to basic knowledge and skills in pure mathematics. It may rather emphasize the pupils' need to be able to orient themselves in the world of pure mathematics as a necessary prerequisite to solving real-world problems. Several articles based on data from TIMSS and PISA have pointed out that one of the most problematic issues in the Nordic countries is that there has been too much focus on applied mathematics and too little on basic knowledge in pure mathematics, and that this has resulted in low achievement both in TIMSS and PISA (Grønmo, 2010; Grønmo & Olsen, 2006; Olsen & Grønmo, 2006).

Figure 1. The Mathematisation Cycle (Source: (NCTM, 1989))

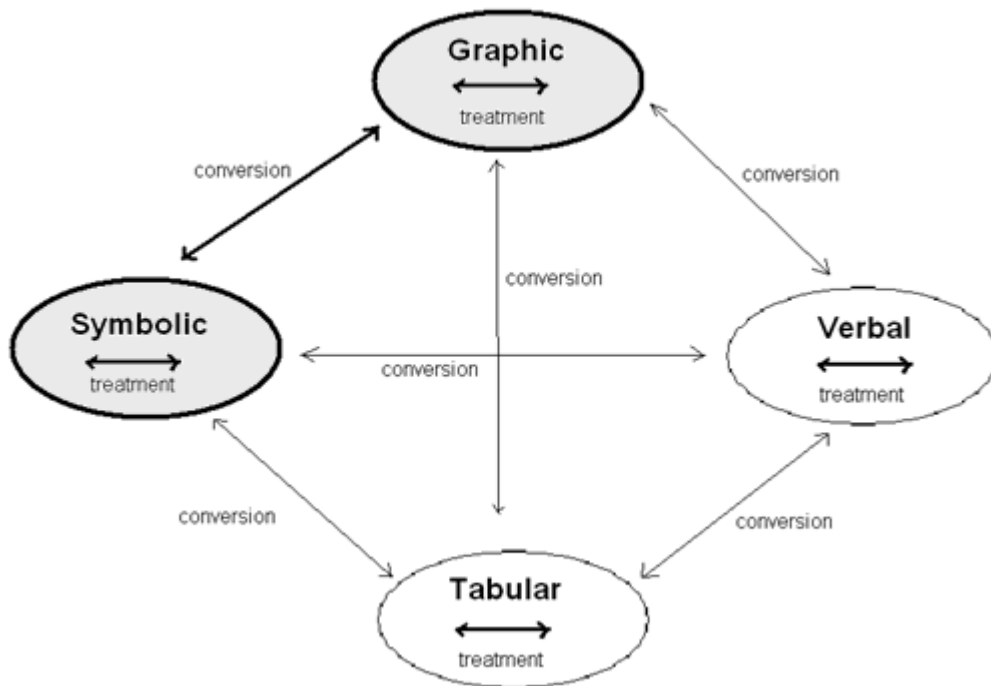


### Representations and translations between these

Algebra has traditionally been associated with symbolic expressions and manipulations of these. However, there has been an increasing interest in the use of graphical representations over the last 20 years or so. This is reflected both in research reports (see for instance (Bardini, Pierce, & Stacey, 2004; Huntley, Marcus, Kahan, & Miller, 2007; Yerushalmy & Gafni, 1992)) and in the fact that official documents such as the NCTM standards recommend that upper secondary school students should be able to “represent and analyze relationships using tables, verbal rules, equations and graphs” (NCTM, 1989, p. 154).

Figure 2 shows the so called “Rule of four”-model of multiple representations. (This is slightly adapted from a rather similar figure presented in a paper by Huntley et al. (2007).) In line with Duval (2006), the “tags” *treatment* and *conversion* have been added to the figure.

Figure 2. The rule of four model of multiple representations. We are mainly focusing on symbolic and graphic representations, and have tried to emphasize this part of the diagram by using shading and thicker lines



Treatments are transformations of representations that stay within the same register, for example solving an inequality with purely symbolic manipulations. Conversions, on the other hand, are transformations where one changes a register without changing the objects being denoted, for example passing from the symbolic formulation of an inequality to a graphical representation of the same inequality. This is, according to Duval, much more cognitively demanding than treatment (Duval, 2006). His view is supported by Sackur (2004), who notes that in order to solve a standard inequality (symbolic formulation) graphically one must do the following work:

Inequality on symbolic form ( $f(x) > g(x)$ )  $\rightarrow$  transform to functional form ( $y = \dots$  and  $y = \dots$ )  $\rightarrow$  draw the graphs  $\rightarrow$  compare the graphs  $\rightarrow$  write down the truth set ( for instance on the form  $a < x < b$ )

In this paper, we will refer to the different cognitive demands of treatments and conversions when discussing Norwegian students' performance in solving inequalities.

## **Methodology**

### **Description of the study and the participants**

This paper is based on data from the mathematics achievement test in TIMSS Advanced. Here, 'Advanced' refers to the fact that this study aims at describing the performance of students participating in the most advanced mathematics and physics courses offered at the upper secondary school level. Ten countries participated in TIMSS Advanced; Armenia, Iran, Italy, Lebanon, the Netherlands, Norway, the Philippines, Russia, Slovenia and Sweden. The age of the assessed students ranged from 16 (in the Philippines) to 19 (in Norway, Sweden, Italy and Slovenia). Furthermore, the number of years of formal schooling varied between students from the different countries; from 10 years of schooling in Armenia and the Philippines, to 13 years of schooling in Italy (Mullis, Martin, Robitaille, & Foy, 2009). There are also large differences between countries when it comes to the *Coverage index*, which is the proportion of students in the actual age cohort that is defined as the population to be tested in TIMSS Advanced. This differs from less than 1% in the Philippines, to over 40 % in Slovenia. Norway defined the population as students taking the most advanced mathematic course in the last year of upper secondary school (3MX), which is 11% of the age cohort in the country (ibid.).

In this paper, the focus will mainly be on the Norwegian students. In Norway, all upper secondary schools offering the most advanced mathematics (3MX) and/or physics (3FY) course were invited to participate in TIMSS Advanced. This led to 120 schools being asked to participate in mathematics, and 118 in physics. Of the 2206 students enrolled in the course 3MX at schools included in the mathematics study, 1932 students (88 %) participated. These students had 12 years of formal schooling (and consequently 12 years of mathematics instruction), and their average age was 19.

### **Selection and categorization of items**

From the mathematics achievement test in TIMSS Advanced, the following two items related to the topic of algebraic inequalities have been selected:

### Item 1

Two mathematical models are proposed in order to predict the return  $y$ , in dollars, from the sale of  $x$  thousand units of an article (where  $0 < x < 5$ ). Each of these models,  $P$  and  $Q$ , is based on different marketing methods.

$$\text{model : } y = 6x - x^2$$

$$\text{model : } y = 2x$$

For what values of  $x$  does model  $Q$  predict a greater return than model  $P$ ?

- A)  $0 < x < 4$       B)  $0 < x < 5$                       C)  $3 < x < 5$   
D)  $3 < x < 4$       E)  $4 < x < 5$

### Item 2

$$\frac{x + 1}{x - 2} > 1$$

For which values of  $x$  is the inequality shown above satisfied?

Answer: \_\_\_\_\_

Basic quantitative analyses of the Norwegian student performance on these items are conducted by calculating frequency tables, and the Norwegian results are further illuminated by making comparisons to international data for the selected items. These items are also qualitatively analyzed, which amounts to describing similarities and differences of the items with respect to surface features (for instance, different contexts or modes of representation) and deeper mathematical structures.

One of our aims is to relate Norwegian upper secondary school students' mathematical performance in solving algebraic inequalities to curricular objectives and central features of school mathematics in Norway. In order to discuss such central aspects of mathematics as a subject in Norwegian schools, we have also categorized all the items from the mathematics achievement test in TIMSS Advanced in terms of *item format* (Multiple Choice/MC or Constructed response/CR) and *item context* (Applied mathematics or Pure mathematics). For each item category, we have calculated an average p-value (average percentage correct on the

items in the category) for the Norwegian students and for the international average on these items. The average p-values are compared in order to shed some light on how student performance is related to item format and item context.

## Results and discussion

Table 1 shows the proportion of students giving a correct solution, incorrect solution and no answer to items 1 and 2. The Norwegian results are here compared to the international average for these items. As the table show, the international average on item1 and item 2 is approximately the same. The performance of the Norwegian students does, however, vary a lot between the tasks; while 59 % of the Norwegian students solved item 1 correctly, only 16 % managed to produce a correct answer to item 2

*Table 1. Distribution of answers for the 2 selected items. Norwegian students compared to the international average*

Item	Type of answer	Norway	International average
Item 1	Correct (alternative E)	59 %	51 %
	Incorrect (alternative A, B, C or D)	34 %	38 %
	No answer	7 %	11 %
Item 2	Correct ( $x > 2$ )	16 %	45 %
	Incorrect	64 %	48 %
	No answer	20 %	7 %

As neither of these tasks required the students to show their work, one cannot immediately explain why the Norwegian students performed so differently on items that both involve solving inequalities, and that internationally were of approximately similar difficulty. However, something can be learned by qualitatively analyzing the items and identifying the mathematical knowledge and skills required to solve the different tasks, as well as relating the items to the curriculum the Norwegian students have followed and other central features of school mathematics in Norway.

Item 1 is a multiple-choice task set in a “real-world” context, where the students are asked to compare two models formulated in terms of a linear and a quadratic function (which is equivalent to solving a quadratic inequality). At first glance, item 2 looks very different from item 1. Item 2 is a constructed-response item, where the students must formulate an answer themselves. It has no “real-word” context, and it is *not* formulated in terms of two functions



that are to be compared. Rather, it is a “pure” algebra problem where the students are asked to solve a rational inequality.

In order to further investigate how student performance may be related to features of the problem presentation, we have categorized all the 71 items from the mathematics achievement test in TIMSS Advanced in terms of *item format* (Multiple Choice/MC or Constructed Response/CR) and *item context* (Applied Mathematics or Pure Mathematics). In table 2, we summarize the students’ performance across the two types of item format and the two types of item context. In this analysis, average percent correct (average p-value) on a set of items is used as a measure of student performance. The table hence shows the average p-value for all of the advanced mathematics items, as well as within each of the four domains (MC, CR, Applied Mathematics and Pure Mathematics). Again, the Norwegian results are further illuminated by making comparisons to international data.

*Table 2. Results (average p-values) in relation to item format and item context.*

	Advanced Mathematics (71 items)	Item format		Item context*	
		MC (45 items)	CR (26 items)	Applied Mathematics (11 items)	Pure Mathematics (60 items)
Norway	33	40	22	39	32
International Average	40	46	30	35	41

\* Items classified as “Applied Mathematics” present problems set in a (simplified) “real world”-context. Items classified as “Pure Mathematics” present problems set in a mathematical context.

In table 2, student performance on all the mathematics achievement items in TIMSS Advanced is included for reference, and we see that the Norwegian students performed below the international average on this test. Here, we are however more interested in how the performance may be related to item format and context. With respect to item format, it is clear that the Norwegian students performed distinctively better on Multiple-Choice items compared to Constructed-Response items, which is similar to the pattern found in the international data. In terms of item context, there is however an interesting difference between the Norwegian and the international data: The Norwegian students performed better on items classified as Applied Mathematics than on items classified as Pure Mathematics, whereas internationally, students were more successful in solving Pure Mathematics-tasks. This is in line with earlier research for comprehensive school based on analyses of both

TIMSS and PISA data (Grønmo, 2010; Olsen & Grønmo, 2006). An analysis of PISA 2003 data revealed that Nordic countries had particularly high achievement on items set in a “real world”-context, while these countries performed lower on tasks requiring formal mathematical knowledge such as exact calculations and use of algebraic expressions (ibid.).

It then seems that the difference in item format and context can in part account for why the Norwegian students performed so differently on the two selected items (item 1 and item 2). It is, however, unlikely that this is the whole explanation. Another perspective can be found by considering the mathematical content of the two items more closely. In this respect, there are some clear similarities between the two tasks: Both involve solving inequalities on the form  $f(x) > g(x)$ , and the same methods can be used in both cases. One possible approach is to solve it graphically, that is, to compare the graphs of the functions  $f(x)$  and  $g(x)$ . Another approach is outlined in the Norwegian curriculum, which states that students should “understand how to simplify equations and inequalities” and “use the sign-chart method to solve quadratic and rational inequalities” (KUF, 2000, p. 9, our translation). Solving inequalities with this approach entails transforming the inequality to the form  $F(x) > 0$ , factoring the expression  $F(x)$ , and using a sign chart to determine the sign of each factor and of the full inequality. As noted in the quotation, this method is to be used both for quadratic and for rational inequalities.

Based on the considerations above, we try to find other plausible explanations for the difference in the Norwegian students’ performance on the two items. Let us first consider a solution using the sign-chart method. Any solution using this method will have to start with transforming the inequality to the form  $F(x) > 0$ , and we will focus on the required algebraic transformations rather than on the sign charts.

Item 1	Item 2
$2x > 6x - x^2$ $2x - 6x + x^2 > 0$ $x^2 - 4x > 0$ $x(x - 4) > 0$	$\frac{x + 1}{x - 2} > 1$ $\frac{x + 1}{x - 2} - 1 > 0$ $\frac{x + 1}{x - 2} - \frac{x - 2}{x - 2} > 0$ $\frac{3}{x - 2} > 0$

One possible reason for the low solution-rate of item 2 compared to item 1 may be that the transformations required to solve this inequality are slightly more complicated (for instance requiring the student to convert a number to a fraction). It may also be the case that a number of students could have tried to solve item 2 as if it is an equation:

$$\begin{aligned} \frac{x+1}{x-2} > 1 & \quad | \cdot (x-2) \\ x+1 > x-2 \\ 1 > -2 \end{aligned}$$

Another possible reason for the difference in the Norwegian students' performance on the two items may lie in the way the tasks are formulated. The use of graphic calculators is integrated in most aspects of the mathematics teaching in Norway, and students learn to draw graphs using the calculator, and to solve equations and inequalities graphically. Both items are formulated in terms of algebraic symbolic expressions. To solve the inequalities graphically will in both cases involve a change of register, a conversion, in the language of Duval. But although conversions are said to be more demanding than transformations within one register (like symbolic manipulations), one may argue that solving item 1 graphically is rather easy compared to solving item 2 in this way. As noted in the theory section, a graphical solution requires that the students do the following work:

Inequality on symbolic form ( $f(x) > g(x)$ )  $\rightarrow$  transform to functional form ( $y = \dots$  and  $y = \dots$ )  $\rightarrow$  draw the graphs  $\rightarrow$  compare the graphs  $\rightarrow$  write down the truth set ( for instance on the form  $a < x < b$ )

(Sackur, 2004).

Item 1 essentially asks the students to compare the graphs of the functions  $y = 6x - x^2$  and  $y = 2x$ , which may be done directly by entering the two expressions into the calculator. Hence, when solving item 1 there is no need to “transform to functional form”. The inequality in item 2 is however not on this form, and according to Sackur (2004), the emergence of  $y$  and its role is often a source of difficulty for students. That is, when solving the inequality  $\frac{x+1}{x-2} > 1$  it is not trivial to realise that this can be done by comparing the graphs of the functions  $y = \frac{x+1}{x-2}$  and  $y = 1$ . This means that although both inequalities can be solved graphically, it is more demanding to solve item 2 in this way.

Since neither of the two selected items required the students to show their work, we can not know if the Norwegian students treated item 2 as an equation rather than an inequality, or

whether they easily solved item 1 graphically using their calculator but failed to realize that the same method could have been used on item 2. However, earlier research has shown that (i) students often fail to recognise the difference between inequalities and equations, and consequently have a tendency to divide or multiply with non-positive factors when attempting to solve inequalities (Tsamir & Almog, 2001), and (ii) using graphical representations are demanding for students, and does not necessarily lead to errorless solutions (Sackur, 2004).

## Concluding remarks

While both the items included in our study are related to the topic of algebraic inequalities, they differ in terms of problem presentation (item format, item context, and formulation of the task). An analysis of all the items in the TIMSS Advanced study implies that, internationally as well as in Norway, students perform better on Multiple-Choice items than on Constructed-Response items. Taking into account that there is a factor of guessing in MC-items, this is reasonable. The Norwegian students however, in opposition to the results internationally, have a rather high achievement on tasks set in a “real-world” context relative to tasks set in a purely mathematical context. This may to some extent account for the difference in the Norwegian students’ performance on the two selected items. Furthermore, this is in accordance with results for Norway in comprehensive school, in grade 4, 8, and 10: “The problematic issues for Norway seem to be that too little focus has been on pure mathematics. This conclusion is based on results from a number of studies, as TIMSS 2003 and 2007, as well as from PISA 2003” (Grønmo, 2010, p. 64).

Furthermore, the two items differed in terms of how the tasks were formulated mathematically; one requiring the students to solve a standard inequality while the other asked them to compare different mathematical models that were given on “functional form”. Item 1, given in functional form, invites to a straightforward solution using a graphic calculator. Graphic calculators are commonly used of Norwegian students in upper secondary school (Grønmo, et al., 2010). In item 2, the students have to transform the inequalities from an algebraic symbolic form to a functional form (or to realize that it can be seen as two functions) to solve it graphically using a calculator. This may, in accordance with Duval (2006) and Sackur (2004), be a more cognitive demanding task. Different problem

presentations that, in terms of the deeper mathematical structure, all involve inequalities, evokes different solution strategies that leads to significantly different student performance.

There are several possible problems that need to be further investigated:

- Norwegian students may not be satisfactorily skilled at basic algebraic activity like transforming symbolic expressions. Too little attention given to pure mathematics in Norwegian schools may be a reason for this.
- Students may not have a clear understanding of how inequalities differ from equations, which also may be linked to the need for more attention given to pure mathematics in the school.
- Students' ability to solve inequalities graphically seems to depend strongly on the problem presentation. This may be may be linked to students use of calculators, that is, to what extent the problem formulation invites to a straightforward solution using calculator or not.
- Research based on methods such as observation and interviews of students when they are solving inequalities may give more information about the issues that have been the focus of this paper.

Hence, this analysis of TIMSS Advanced data has led to new research questions, and illustrates that the rich database from large-scale international achievement studies like TIMSS Advanced can be used to obtain much more information than to tell only how well students in a country perform in mathematics.

## References

- Bardini, C., Pierce, R. U., & Stacey, K. (2004). Teaching Linear Functions in Context with Graphics Calculators: Students' Responses and the Impact of the Approach on Their Use of Algebraic Symbols. *International Journal of Science and Mathematics Education*, 2(3), 353-376.
- Bazzini, L., Boero, P., & Garuti, R. (2001). Moving symbols around or developing understanding: The case of algebraic expressions. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th PME International Conference* (Vol. 2, pp. 121-128).

- Bazzini, L., & Tsamir, P. (2004). RF02: Algebraic equations and inequalities: Issues for research and teaching. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*.
- Boero, P., & Bazzini, L. (2004). Inequalities in Mathematical Education: the need for complementary perspectives. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th PME International Conference* (Vol. 1, pp. 137-139).
- Dobbs, D. E., & Peterson, J. C. (1991). The Sign-Chart Method for Solving Inequalities. *Mathematics Teacher*, 84 (8), 657-664.
- Dreyfus, T., & Eisenberg, T. (1985). A Graphical Approach to Solving Inequalities. *School Science and Mathematics*, 85(8), 651-662.
- Duval, R. (2006). A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics. *Educational Studies in Mathematics*, 61(1), 103-131.
- Gardiner, A. (2004). *What is mathematical literacy?* Paper presented at the ICME 10, Copenhagen.
- Garuti, R., Bazzini, L., & Boero, P. (2001). Revealing and promoting the students' potential: A case study concerning inequalities. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th PME International Conference* (Vol. 3, pp. 9-16).
- Grønmo, L. S. (2010). Low Achievement in Mathematics in Compulsory School as Evidenced by TIMSS and PISA. In B. Sriraman, C. Bergsten, S. Goodchild, G. Pálsdóttir, B. Dahl & L. Haapasalo (Eds.), *The First Sourcebook on Nordic Research in Mathematics Education* (pp. 49-69). Charlotte, NC: Information Age Publishing.
- Grønmo, L. S., Kjærnsli, M., & Lie, S. (2004, May 11-13). *Looking for Cultural and Geographical Factors in Patterns of Responses to TIMSS Items*. Paper presented at the 1st IEA International Research Conference (IRC-2004) Lefkosia, Cyprus.
- Grønmo, L. S., & Olsen, R. V. (2006, November 9-11). *TIMSS VERSUS PISA: The Case of Pure and Applied Mathematics* Paper presented at the 2<sup>nd</sup> IEA International Research Conference (IRC-2006), Washington D.C, United States.
- Grønmo, L. S., Onstad, T., & Pedersen, I. F. (2010). *Matematikk i motvind-TIMSS Advanced 2008 i videregående skole*. Oslo: Unipub.
- Huntley, M. A., Marcus, R., Kahan, J., & Miller, J. L. (2007). Investigating high-school students' reasoning strategies when they solve linear equations. *Journal of Mathematical Behavior*, 26, 115-139.
- Knuth, E. J. (2000). Student understanding of the Cartesian Connection: An exploratory study. *Journal for Research in Mathematics Education*, 31(4), 500-508.

- KUF (2000). *Læreplan for videre opplæring (Matematikk 2MX/3MX)*.
- Mullis, I. V. S., Martin, M. O., Robitaille, D. F., & Foy, P. (2009). *TIMSS Advanced International Report*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.
- NCTM (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA.
- Olsen, R. V., & Grønmo, L. S. (2006). What are the Characteristics of the Nordic Profile in Mathematical Literacy? In J. Mejdning & A. Roe (Eds.), *Northern Lights on PISA 2003 - a reflection from the Nordic countries* (pp. 47-57). Copenhagen: Nordic Council of Ministers.
- Sackur, C. (2004). Problems related to the use of graphs in solving inequalities. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 151-165).
- Tsamir, P., & Almog, N. (2001). Students' strategies and difficulties: the case of algebraic inequalities. *International Journal of Mathematical Education in Science and Technology*, 32(4), 513 - 524.
- Yerushalmy, M., & Gafni, R. (1992). Syntactic manipulations and semantic interpretations in algebra: The effect of graphic representation. *Learning and Instruction*, 2(4), 303-319.