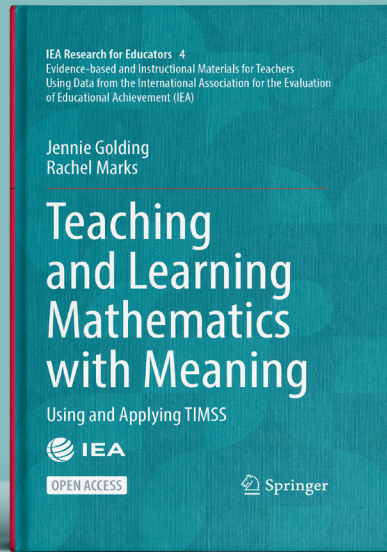




Chapter 3

Highlighted

**Meaningful Mathematics Learning
in Schools**

*Responses to TIMSS That Support
Meaning-Making: Inspiring Examples
From Around the World*

Chapter 3

Responses to TIMSS That Support Meaning-Making: Inspiring Examples From Around the World

Abstract

This chapter outlines the range of influences that the Trends in International Mathematics and Science Study (TIMSS) frameworks and findings have across mathematics education policy and practice. It draws in particular on the *TIMSS 2023 Encyclopedia* and the TIMSS 2023 Mathematics Framework to understand the reasons for that impact. It then focuses on more specific enactments at the classroom level in six education systems across the world. This is examined for learners in Grades 1 to 11 through case studies of individual teachers' practices. The chapter shows how various TIMSS resources are directly or indirectly related to classroom teaching and learning of mathematics. In particular, it highlights efforts to enhance learners' opportunities to learn mathematics with meaning. In doing so, the chapter also indirectly draws attention to the importance of context for the implemented curriculum, the limitations of policy borrowing, and the influence of other key variables affecting learners' opportunity to learn. These variables include teachers' mathematical and pedagogical preparation, their interpretations of national policy, the instructional time allocated to in-school mathematics learning, and the approaches adopted for its delivery.

Keywords

impact of TIMSS; opportunity to learn; policy borrowing; Trends in International Mathematics and Science Study (TIMSS)

3.1 How Do Countries Respond to TIMSS Findings About Education Systems, Performance, and Learner Experiences?

Countries participate in the International Association for the Evaluation of Educational Achievement (IEA) Trends in International Mathematics and Science Study (TIMSS) for a variety of reasons. Given the significant financial and human resource investment required for such large-scale studies, understanding how their findings are used is particularly important. This chapter exemplifies the influence of TIMSS data and frameworks on policy and classroom practice across diverse education systems and with varying levels of educational resourcing.

It should be noted that in early cycles learners from the global south experienced comparatively low rates of success, which is likely to have limited the influence of TIMSS data and frameworks in those education systems. In response, TIMSS assessments have recently been redeveloped to be more adaptive and more inclusive of a wider range of

attainment. Further, IEA has also established the Literacy and Numeracy Assessment (LaNA), a new reading and mathematics assessment on which a wider range of learners can confidently demonstrate what they know, understand, and can do. LaNA provides standardized, reliable data on the strengths and weaknesses of a range of education systems. It also addresses the global demand for comparative data to measure progress toward the Sustainable Development Goals, as well as the need for capacity building in educational research. Further details can be found on the IEA website¹.

One obvious benefit of participating in TIMSS or other international large-scale assessments is that a country can see how their learner attainment in mathematics and science compares with that in other countries. They may select appropriate comparators, such as those with similar cultural contexts. Findings, however, must be interpreted with caution. For example, if a country's curriculum does not include significant parts of the relevant TIMSS framework for a given grade, it would be unreasonable to expect learners to perform well in those areas. Participating education systems therefore conduct a curriculum-matching analysis of their dominant or intended curriculum for the participating grades against the relevant TIMSS framework, and this analysis is reported in each cycle's TIMSS encyclopedia. It enables users of TIMSS data to evaluate how well aligned each TIMSS framework—and, by extension, each TIMSS assessment—is with the typical curriculum experience of a Grade 4 or Grade 8 learner.

For each participating education systems, IEA provides analyses of average performance across all assessment items, as well as average performance on items typically encountered by learners at that stage. We note, though, that average mathematics performance levels in mathematics are not highly correlated with TIMSS curriculum-framework alignment. For example, some of the highest-performing education systems have curricula that are relatively less aligned with the TIMSS framework in terms of content coverage. As with all comparative analyses, the power of such approaches lies in the questions they enable us to ask as we seek to understand differences in performance. In this case, it appears that learners in such education systems are achieving a deeper and more robust grasp of mathematical knowledge, application, and reasoning: they are showing a strong sense of mathematics with meaning, in the sense developed in Chapters 1 and 2.

As we outlined in Chapter 1, TIMSS context questionnaires also provide substantial data around learner experiences, home and school contexts, and teacher practices and perceptions, with some countries also assessing parental perspectives. The questionnaire data are essential for analyzing not only what learners know, understand, and can do, but also their attitudes, aspirations, and confidence to continue engaging with mathematics and science. These findings can also be used for comparative purposes, although again, there are important limitations to their interpretation. Survey responses

¹ See <https://www.iea.nl/studies/iea/LaNA>.

are both subjective and influenced by cultural norms and expectations. For example, both teacher and learner reports of disruptive behavior are likely to be interpreted very differently in cultures where quiet classroom conformity is the norm than in countries or systems where classroom interaction is more openly expressed.

National interpretation of comparative findings can be further informed by reference to the TIMSS encyclopedia, which outlines countries' education systems. Several instances of the importation of educational approaches are discussed later in this chapter. However, there is substantial evidence showing that policy borrowing is not straightforward: the impact of a particular policy varies significantly, depending on the characteristics of both the original and importing education systems, as well as their local cultures and contexts (Burdett & O'Donnell, 2016). Participating countries vary in the extent and ways in which they directly harness TIMSS data to inform policy, and such use may, in some cases, serve to affirm the continuation of existing policies. Others might use TIMSS data to challenge and further develop recommended mathematics and science education practices. Of course, the ways in which schools and teachers then make local adaptations of policies or recommended practices can also vary significantly (see, for example, Spillane, 2004); equally, we know it is important that teachers feel they have some agency to adapt to the needs of their own classrooms (see, for example, Priestley et al., 2015). Taken together, then, it is preferable that teachers understand the rationale for reforms that are proposed in the light of evidence such as TIMSS, but it is neither feasible nor desirable that all, for example, Grade 4 mathematics classrooms in a given country appear to function in exactly the same way: each classroom features a unique teacher, a unique set of learners, and functions in a unique context.

3.1.1 Examples of National Responses to TIMSS

Jordan, for example, has made extensive and varied use of their TIMSS data over many years: as a country with very limited natural resources, Jordan has been keen to develop its education system, and for over 25 years the Ministry of Education has invested in providing high-quality curricula, textbooks, and teacher manuals. These have been further developed to emphasize critical and creative thinking, problem-solving skills, and the linking of subject content to real-life experiences. Their implementation has been supported by the introduction of a 2015 national policy on teaching and learning materials. In mathematics, there has been a particular focus on communication and information management skills. In addition, instructional materials are designed to help learners apply information in their academic and everyday lives (Ababneh et al., 2016).

Jordan's natural comparator countries are the other Arab education systems, in relation to whom Jordan's learners have often performed relatively well. However, TIMSS findings have supported critiques of Jordan's education systems within a broader comparative context, contributing to further curriculum reforms. These reforms have included the provision of sustainable, smart electronic learning resources and an increased emphasis

on improving the quality of electronic services provided by the Ministry. Released TIMSS items from all previous cycles (1999, 2003, 2007, 2011, 2019, 2023) have been used in the development of new textbooks for mathematics and science (Ababneh et al., 2024). The impact of such changes on one teacher is evidenced in case study 2 (Section 3.3).

In a quite different context, **England** has in recent years explicitly sought to transfer some of the approaches to mathematics teaching and learning used in two consistently high-performing education systems in large-scale assessments: Singapore (TIMSS) and Shanghai (the OECD Programme for International Student Assessment [PISA]). Central authorities have established a network of Maths Hubs² tasked with supporting local teachers in developing teaching for mastery, an approach that synthesizes several of the identified distinctive practices.

Other education systems respond to TIMSS data in a variety of ways at the national level, including using data to affirm their current policies and recommended practices. Of course, an individual teacher's response in classrooms is influenced by policy in very complex ways, and the relationship between those influences and TIMSS data or frameworks is equally complex. We explore these relationships in Sections 3.1.3 and 3.2 onwards. First, however, we outline the TIMSS mathematics frameworks to show how they relate to both common and emerging curricula across the globe.

3.1.2 TIMSS Mathematics Frameworks and Meaning-Making

TIMSS mathematics assessments are focused on not only foundational mathematics skills but also mathematical reasoning, problem-solving, and real-world applications. As we explored in Chapters 1 and 2, these are integral to learning mathematics with meaning, with problem-solving approaches and processes assessed through more complex, interactive items. Assessments are developed to align with the TIMSS mathematics framework, available for each cycle on the IEA TIMSS & PIRLS website³. The framework, while relatively stable in order to understand trends over time, also evolves over time to reflect current and emerging educational priorities in mathematics. For example, the TIMSS 2027 Mathematics Framework (Philpot & Aldrich, 2025) was developed to incorporate a specific focus on computational thinking for the first time.

Each TIMSS mathematics framework also includes a referenced rationale for the importance of learning mathematics in ways that empower and equip learners for making greater meaning of their dynamic worlds.

The TIMSS mathematics frameworks are structured around *content* and *cognitive* (process) domains, with each assessment item classified within one content domain (number,

² See <https://www.ncetm.org.uk/maths-hubs/>.

³ See <https://timss.bc.edu/timss-landing.html>.

measurement and geometry, and data) and one cognitive domain (knowing, applying, and reasoning). The Grade 4 mathematics assessment places a greater emphasis on topics in the number content domain than on either measurement and geometry or data, reflecting similar curricular emphases internationally and including introductory algebra (or pre-algebra) topics. Grade 8 assessments, reflecting common curriculum practice internationally, include algebra as a stand-alone content domain, alongside number, geometry and measurement (now with a focus on geometry), and data and probability.

Generally, the cognitive skills understood to underpin mastery of subject matter progress from developing a knowledge base to applying that knowledge in unfamiliar contexts to making judgments or justifying arguments. These stages are reflected in the TIMSS cognitive domains and are all essential for problem-solving. As is common, though not universal, internationally, TIMSS uses problem-solving to mean a process directed at achieving a goal when the learner does not initially know a solution method. Problem-solving in mathematics then takes place either in real-world or purely mathematical contexts. In TIMSS items, and in problem-solving more generally, the extent of familiarity with the problem context can influence the level of cognitive engagement required to successfully address the problem. TIMSS items typically assess only selected stages of the problem-solving process. However, within the assessment, Problem-Solving and Inquiry items (PSIs) are embedded in real-world tasks and offer a more extended measurement of problem-solving. These items include multiple steps of problem-solving and higher-order cognitive skills across several related items situated within more complex scenarios. As a result, PSIs are able to assess target content and cognitive domains more deeply and authentically, thereby strengthening opportunities for mathematical meaning-making.

TIMSS data support the understanding of educational opportunities and outcomes across education systems via the TIMSS Curriculum Model (Mullis & Martin, 2017). The TIMSS Curriculum Model comprises three interconnected aspects: the *intended*, *implemented*, and *attained* curriculum. The intended curriculum encompasses contexts that include system-level policies, curricular standards, and organizational structures designed to facilitate learning; these are captured in each cycle's encyclopedia. The implemented curriculum covers contexts surrounding the translation of curricula into practice, including instructional delivery, teacher practices, and home and school environments; these are evidenced through rich background questionnaires completed by learners, parents, teachers, and school leaders. The attained curriculum focuses on learner attainment, assessed for mathematics in line with the framework analyzed above, and their attitude, captured via learner questionnaires. The data, taken together, support analysis of how education systems and contexts shape learner outcome, and, in particular, the extent to which they are succeeding in learning mathematics with meaning.

In the case studies that follow, we outline at a high level the policies and structures within which teachers work while also describing key aspects of the immediate physical,

organizational, and didactical contexts in which their learners engage. We highlight the ways in which individual teachers’ professional knowledge and practices shape learners’ opportunities to learn mathematics with meaning and identify points of alignment with the TIMSS 2023 Mathematics Framework (Philpot et al., 2021).

3.1.3 Case Studies of Meaning-Making in Classroom Practice Across the Globe

The main focus of this chapter is not on which TIMSS-catalyzed policy changes have been introduced, but on how those changes—and the TIMSS frameworks—are reflected in classroom practices that support meaning-making. Sections 3.2–3.7 present six case studies, spanning Grades 1 to 9, drawn from education systems where TIMSS findings or frameworks have at least somewhat influenced changes in curriculum, teaching approaches, and/or assessment (as evidenced in the *TIMSS 2023 Encyclopedia*, Reynolds et al., 2024). Each case examines how such influences are enacted in the practice of an individual teacher in that education systems.

While we do not claim that any of these teachers is typical of those in their education systems, their stories celebrate the international efforts that are being made to enhance the quality and enjoyment of mathematics education made available to learners. They also serve to stimulate reader reflection on what matters in mathematics classrooms and the range of approaches that teachers can take to promote confident and successful engagement with mathematical thinking so that learners come to make genuine meaning of the mathematics they encounter.

The education systems represented are Chinese Taipei, Jordan, Türkiye, Sweden, Malta, and Malaysia. Overall, we sought case studies from a wide range of contexts and different grade levels so as to exemplify a range of approaches to teaching mathematics with meaning. The cases are presented in order of learners’ increasing levels of experience. [Table 3.1](#) summarizes the distinct focus of each case study.

Table 3.1: Case studies of meaning-making in mathematics classrooms

	Education system	Focus and grade level
Case study 1	Chinese Taipei	Addressing divides between rural and urban schools in Grades 1 and 3
Case study 2	Jordan	Using TIMSS-focused teacher guides in Grade 4
Case study 3	Türkiye	Enhancing strong foundations for attainment in Grades 5–8
Case study 4	Sweden	Teaching mathematics for meaning with a focus on language in Grades 7–9
Case study 5	Malta	Shifting toward more learner-focused approaches in Grades 8–10
Case study 6	Malaysia	Embracing higher-order thinking skills in Grades 10 and 11

Note. Country-level data were sourced from the TIMSS 2023 Encyclopedia. Teachers provided questionnaire narratives describing their approaches to teaching, together with a typical example, and reflected on the reasons for adopting those approaches. Teachers’ names and schools are included only where local consent was obtained.

3.2 Case Study 1: Addressing Divides Between Rural and Urban Schools in Chinese Taipei in Grades 1 and 3

TIMSS-related developments in Chinese Taipei

Chinese Taipei has participated in TIMSS since 1999. The results have been used to evaluate the effectiveness of mathematics and science education. While early results validated the success of curriculum guidelines for Grades 1–9 in terms of outstanding achievement, subsequent TIMSS series exposed persistent challenges, including learners’ low interest in mathematics compared to other countries, a notable proportion falling below the Intermediate International Benchmark of mathematics achievement, wide rural-urban achievement gaps, and less engaging teaching in Grade 8 mathematics. These issues have prompted attention from educators and policymakers, catalyzing substantial reforms, including the 2017 Act for Education Development of Schools in Remote Areas. The associated support in the Act has since been extended to learners with low achievement in mathematics. In addition, the Ministry of Education has sponsored summer camps for learners in rural areas to help reduce the achievement gaps between urban and rural learners.

Although average TIMSS mathematics performance in Chinese Taipei is one of the strongest internationally, it is important to notice the efforts that are being invested to make sure high-quality learning is available to the range of learners, regardless of where they live within the country. This case study focuses on the ways the teacher sets out to connect mathematical ideas with learners’ everyday experiences, supporting them with making meaning of the mathematics they are using and succeeding in the task. At different stages, she overtly supports reasoning, application, and positive attitudes, as well as

foundational mathematical knowledge. As is typical with genuine problems in a real-world context, she draws on a number of the TIMSS content and cognitive domains to help the learners in her class make meaning of the mathematics they are using.

School, Subject, and Teacher Context

Han Hui (a pseudonym) teaches Grades 1 and 3 in a rural elementary school. Learners in Grades 1–9 receive 160 minutes of mathematics instruction a week in four sessions; Han Hui teaches two of these weekly sessions for each of Grades 1 and 3. She completed her teacher education at Teachers College, graduating from the Department of Education. She majored in mathematics education in the master’s program of the graduate school, and majored in curriculum and teaching in the doctoral class.

Han Hui’s approach is to build bridges of understanding through teaching aids, deepen the essence of knowledge through mathematical thinking, and attach importance to the cultivation of mathematical affective aspects so that learners can learn mathematics in a fun and supportive environment. Her experience is that through step-by-step guidance and appropriate challenges, learners can experience the joy of success, build self-confidence, and believe that they can solve mathematical problems. She encourages learners to ask questions and gives timely feedback, encouraging their efforts and progress and enhancing their positive feelings toward mathematics.

Han Hui focuses not only on learners getting the correct answers but also on cultivating their logical and critical thinking. She encourages learners to think about “Why?” and “Is there another way?”, prompting them to explain their reasoning processes. Mistakes are treated as opportunities for learning, and learners are supported in applying mathematical knowledge flexibly across different situations and in developing independent thinking skills. She believes that through assignments, games, and group interactions, learners can feel that mathematics is not just a number on the exam paper but also a language to explore the world. Han Hui finds this kind of sensory engagement fosters learners’ interest in mathematics and gives them a sense of accomplishment and satisfaction in their learning: from the study of mathematics courses, learners will be able to analyze problems and make inferences, which is helpful in their future life and work. Therefore, she believes teachers should emphasize the value of mathematics in teaching and help learners build a long-term interest and love for mathematics.

In recent years, with the continuous development of the national curriculum and particular support for rural areas, Han Hui feels that her teaching philosophy and practice have also changed—mainly through a focus on learners’ mathematical language.

She encourages learners to express their ideas in natural language, rather than quick formulas or procedural strategies to “come up with answers.” Her aim is to deepen their understanding of core concepts through descriptions, arguments, and communication. She has also adjusted the learning methods in her classroom. Learners are permitted to make mistakes in their thinking and writing and are encouraged to spend more time exploring problems in depth. She appreciates learners’ ideas in the learning process and uses teaching aids to concretize abstract mathematical concepts and establish intuitive feelings through the five senses so that learners cultivate their ability to do and understand mathematical ideas. Overall, she now places greater emphasis on the development of mathematical thinking. Han Hui gives the following as an example:

Teaching objective: To allow learners to construct the concept of speed through their own experience of related situations.

Activity content: Each child records the travel time spent from home to school for four consecutive days and the mode of transport. One of the days is selected to organize the whole class data table (Table 3.2). Learners’ distance from home to school in meters is measured by using Google Maps, and the time is measured to the nearest minute (note the use of digital technology to enhance mathematical activity with meaning).

Task 1: The class observes and discusses the class statistics table (real data measured by the whole class), exploring what might affect the speed of getting from home to school.

Task 2: Learners compare who is faster and who is slower among the four learners who walk to school.

Table 3.2 Data for walking journeys

Class seat number	Vehicle	Length of road from my house to school (meters)	December 10 Time spent (minutes)
1	walk	350	3
4	walk	400	5
7	walk	350	5
8	walk	350	3

This data provides learners with the opportunity to explore and solve speed problems.

Focus question: How do you compare different road lengths and times (for example, 350 m in 3 minutes and 400 m in 5 minutes)?

Six groups of learners suggest four different strategic approaches to compare and analyze relationships over time and distance (Table 3.3).

Table 3.3 Group strategies for comparing journeys

	Two groups	One group	One group	Two groups
Strategy	Fix the distance and compare the time taken	Fix the time and compare the distance traveled	Compare the distance traveled in 15 minutes (a common multiple of 3 and 5)	Compare the distance traveled per minute

The strategies are discussed and compared by the class.

Task 3: Compare the strategies of three different scenarios to compare who is fast and who is slow.

Han Hui’s learners share their data and discuss “Who is faster and who is slower when completing the same journey from home to school?” and learn from each other in the process. Through these discussions, learners are able to examine their own myths and misconceptions. They gradually revise and refine their understanding through peer interaction, in line with her beliefs around the importance of building on what learners already know. This interaction not only deepens their understanding of the concept of speed and a range of related ideas but also enhances their interest and engagement in learning.

Didactic principles in Han Hui’s classroom:

- **Teaching responds to rural education priorities in Chinese Taipei through engaging teaching and active learner participation.**
- **Teaching aids and multiple representations are used strategically** to build conceptual bridges of understanding and support deeper mathematical meaning-making.
- **Conceptual understanding is strengthened through step-by-step guidance and appropriate challenges**, enabling learners to develop mathematical thinking.
- **Importance is given to the affective aspects of learning mathematics**, allowing learners to experience the joy of success, build self-confidence, and believe that they can solve mathematical problems.



3.3 Case Study 2: Using TIMSS-Focused Teacher Guides in Jordan in Grade 4

TIMSS-related developments in Jordan

Building on TIMSS analyses as described in Section 3.1.1, the Ministry of Education in Jordan developed teacher guides and initiated nationwide discussions and teacher training to raise awareness of the importance of the TIMSS assessment and its findings. These teacher guides are being used in teacher education programs, as well as classrooms, and appear to have had a positive effect on learner achievement in science and mathematics. The teacher guides include the identification of common assessment errors and misconceptions, analyses of their likely origins, and suggested teaching and learning strategies to support both teachers and learners in understanding, identifying, and resolving them. These strategies include specific questions and tasks designed to expose these misunderstandings.

In the case study below, the teacher uses a variety of representations and instructional approaches to build deep mathematical meaning-making in relation to large numbers. He seeks to develop not only learners' core knowledge but also reasoning skills and their ability to apply these in varied contexts. He also prioritizes learner engagement and motivation, especially through the inclusion of topic-focused games.

School, Subject, and Teacher Context

Mustafa Jamil Abdullah Mahmoud teaches in an urban school in Amman that provides for learners from age 9 to 18 (Grades 4–12), with classes typically comprising 30–40 learners. He holds a Bachelor of Arts in Mathematics and has 30 years of classroom experience. In addition to his teaching role, he serves as a national mathematics curriculum developer and is a member of the National Team for Computing Mathematics. The learners in his classes receive five 40-minute mathematics lessons per week; he teaches Grade 4 and 9 classes.

Mustafa prioritizes establishing learners' basic concepts and skills that will accompany them throughout their education and life. The Jordan primary curriculum focuses on developing a deep understanding of the four arithmetic operations and the concepts of fractions and decimals, as well as the basics of geometry and measurement. These skills form a solid foundation for subsequent learning in mathematics and contribute to the development of logical thinking and problem-solving.

Mustafa says the TIMSS-based teacher guides are a valuable tool for teachers, providing them with insights into their learners' performance in mathematics and science at the national and international levels. These guides help teachers understand their learners' strengths and weaknesses and identify areas for improvement in curriculum and teaching methods. Even though he is experienced, Mustafa frequently uses the Grade 4 teacher guide to supplement his lesson planning. The guide suggests focused TIMSS tasks that prompt discussion and help identify errors or misconceptions in his classrooms, as well as reminding him of productive questions he can use to formatively assess his learners' understanding.

Mustafa cites the example from a Grade 4 lesson focused on reading and writing large numbers. This lesson addresses reading and writing numbers (in words and numerals) and developing an understanding of place value up to the hundred thousands. He considers this content very important and believes it needs to be approached from multiple perspectives so that learners develop confidence in the component concepts. He therefore includes real-life examples, for instance:

- How would you read the number 574,926 on the license plate of your broken-down car?
- What would be the same, and what would be different, if it were a mobile phone number? Or a landline number of people in Amman?

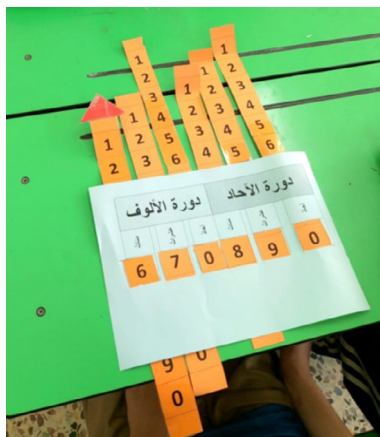
Mustafa makes flexible use of a place value chart that organizes numbers into ones, tens, hundreds, thousands, ten thousands, and hundred thousands. In conjunction with this, he employs a range of resources—colored cards, cubes, and beads labeled with digits from 0 to 9—placed on the chart to represent different place values. He displays numbers on the chart and asks learners to read them aloud as loudly as possible. The faster they read, individually, in small groups, or as a whole class, the better. His learners enjoy competition, so faster and louder is better—provided it's accurate.

He then asks learners to write down large numbers in words when presented in numerals and then reverses the process by providing numbers in words for learners to write in numerals. Learners further develop these skills in pairs or small groups, adopting different roles within the group. He finds that learners particularly enjoy being able to set their own questions, and the breadth and depth of these questions also provide him with formative insight into the conceptual grasp and confidence of the question-setter.

Mustafa's experience is that learners deepen their conceptual understanding through the use of varied visual representations, including:

- Charts or presentations to display numbers visually (see [Figure 3.1](#)).
- Colorful examples to distinguish digits in different place values.
- Games and interactive activities to increase motivation and engagement.

Figure 3.1 Place value manipulative in Mustafa's classroom



For example, in a “missing number” game, Mustafa uses cubes, colored paper, or beads to represent numbers and has learners add or remove items to find the missing number, rather than relying solely on rote memorization. He also creates simple competitive games, of varying difficulty, in which learners read and write numbers to score points against other groups.

Mustafa also uses interactive presentations or apps to visualize large numbers, finding that digital representations can greatly enhance learners’ understanding by showing numerical structure in an engaging way.

He makes consistent use of formative assessment to gauge learners’ conceptual grasp, for example, through the following:

- Direct questions (for example, “What is the value of the number 6 in the number 673,492?”).
- Interactive assessment using digital tools such as Kahoot! and other interactive classroom activities (see [Figure 3.2](#)).

Figure 3.2 Example of an interactive classroom activity

Read the following numbers aloud and then write them in words in the space provided (or vice versa!):

☐ 432,987 → Answer in words: _____

☐ 1,506,132 → Answer in words: _____

☐ 59,834 → Answer in words: _____

☐ 200,005 → Answer in words: _____

Mustafa frequently uses red and green cards to enable learners to self-assess their understanding. Following activities, he reinforces learning by using notes of the mistakes made in class (to be addressed in subsequent lessons) and offering small rewards such as stickers.

Summative end-of-lesson assessment typically involves individual small tasks or worksheets. At times, he might ask learners to design a poster highlighting what they have learned so that all are involved in the development of the classroom learning environment.

Before learners leave the lesson, he reviews the main concepts, either through direct questioning or in a more fun or competitive way:

- Have learners understood place value?
- Are they now able to read large numbers (up to 6 or 7 digits)?

He also invites learners to raise questions and revisits any areas they found difficult, if necessary.

Didactic principles in Mustafa's classroom:

- **Grade 4 mathematics focuses on laying mathematical and attitudinal foundations** for subsequent mathematical learning.
- **All learners are regularly provided with opportunities to develop logical thinking and problem-solving skills.**
- **Competition, variety, and fun approaches are used strategically** to enhance motivation, engagement, and conceptual understanding.
- **Conceptual understanding and the ability to communicate mathematical thinking are prioritized**, with ongoing assessment by both the teacher and the learners valued more highly than producing correct answers.



3.4 Case Study 3: Enhancing Strong Foundations for Attainment in Türkiye in Grades 5–8

TIMSS-related developments in Türkiye

TIMSS is one of several international indicators used to monitor education in Türkiye. Educators consider their learners' scores and rankings on international assessments to be important reflections of their teaching. TIMSS and other international research projects therefore affect curriculum development and education reforms, at least indirectly. The Türkiye Century Education Model (Yüzyılı Maarif)¹ has been developed

¹ See <https://tymm.meb.gov.tr/>.

with an understanding that reflects the sensitivity of the Turkish National Education System to the digital age and technological developments and the desire and potential to lead these developments internationally when necessary. The national education system is comparatively centralized, with a deep-rooted history, and it aims to support the holistic development of the individual learner. High-level thinking skills such as critical thinking, problem-solving, reasoning, and interpretation through real-world scenarios are highly valued in Türkiye. These competencies are monitored through provincial-level assessments administered to Grade 4, 8, and 10 learners.

This case study features a teacher who plans her mathematics teaching with attention to the broader aims of the national curriculum, which emphasizes problem-solving, critical thinking, collaboration, and the application of mathematics across subject areas and in everyday life. Within core knowledge formation, she focuses on depth of concept so that key ideas are built robustly and flexibly and then acquire further meaning through application, reasoning, and problem-solving. This means that she often crosses boundaries between different TIMSS framework cognitive and content domains, but she sees this as central to building meaning-making and positive attitudes.

School, Subject, and Teacher Context

Busra Bozanoğlu works as a mathematics teacher in a secondary school in a rural area of Ankara, the capital of Türkiye. She has 18 years of experience in her role, teaching learners from Grade 5 to 8. For each level of class, five of the 35 hours of weekly lessons are allocated to mathematics. Five mathematical field skills are important for Türkiye: mathematical reasoning, mathematical problem-solving, mathematical representation, working with data and data-based decision-making, and working with mathematical tools and technology. Skills in mathematical representation and data-based decision-making are built on integrated competencies that are specific to the discipline of mathematics.

In the secondary school mathematics curriculum, the basic elements have been shaped according to the principles, approaches, and components adopted by the Türkiye Century Education Model. In this model, mathematics lessons are expected to be planned with a learner-centered, value-oriented, and skill-based approach. This model prioritizes the development of problem-solving, critical thinking, and collaboration skills, while also demonstrating how mathematics is integrated into daily life through technology use and interdisciplinary connections. Assessment is process-oriented and employs diverse methods, and teaching is ideally flexible and differentiated to accommodate individual learner differences.

When planning a specific lesson or a section within a course, **Büşra** ensures it focuses on learners making sense of a mathematical idea, solving a problem, or learning to reason mathematically within this framework. The Ministry of National Education provides a comprehensive set of resources² that teachers in the school use in their lesson planning and assessment, with in-class activities organized by theme. Since they follow a process-oriented approach, Büşra and her colleagues try to give immediate feedback to address learning errors or misconceptions they identify in their learners' work.

The Türkiye Century Education Model offers high-level guidelines for planning mathematics lessons, prioritizing core principles such as learner-centeredness, value-orientation, skill-building, and flexibility. In her classroom, Büşra begins by conducting initial assessments (for example, pre-tests, short interviews, and observations) to determine learners' existing knowledge, strengths, and weaknesses in relation to mathematical concepts. She will typically try to make the lesson more engaging by integrating learners' individual interests and everyday experiences into mathematics tasks. The learning objectives focus on the development of mathematical thinking skills, problem-solving strategies, and analytical approaches, as well as targeting important knowledge. She structures lessons to progress gradually from simpler to more complex concepts based on the learners' levels. Büşra believes learners will learn more deeply and effectively if lessons are enriched with diverse teaching methods such as discovery learning, project-based learning, and collaborative learning.

The Türkiye Century Education Model therefore promotes an expansive view of learning, supported by teacher preparation and access to appropriate resources. TIMSS 2023 results indicate that Turkish learners are thriving in the present system, with mathematics performance at both Grade 4 and 8 comparatively strong in international terms.

² See <https://odsgm.meb.gov.tr/www/5-sinif-matematik/icerik/1275>.

**Didactic principles in Büşra's classroom:**

- **Mathematics lessons are planned with a richly varied, learner-centered, value-oriented, and skills-based approach.**
- **Problem-solving, critical thinking, and collaboration skills are prioritized**, with attention to connecting mathematics to everyday life through technology integration and interdisciplinary connections.
- **Assessment is process-oriented and employs diverse methods**, and instruction is flexible and differentiated to support and challenge individual learners.
- **Immediate feedback is provided** to address emerging errors and misconceptions.
- **Each phase of learning focuses on learners making sense of a mathematical idea, solving a problem, or learning to reason mathematically.**

3.5 Case Study 4: Teaching Mathematics for Meaning With a Focus on Language in Grades 7–9 in Sweden

TIMSS-related developments in Sweden

Sweden's results in TIMSS and other international and national assessments have indicated a decline in average learner attainment, and they have influenced public and professional debate, contributing to significant reforms. These reforms have included enhancing the professional development available to all teachers of mathematics, promoting teaching for conceptual understanding, and paying particular attention to language, especially in response to linguistic diversity within the school population.

The education system in Sweden is comparatively decentralized. Nevertheless, between 2012 and 2016, Sweden ran an extensive and well-received in-service training initiative (Matematiklyftet Skolverkets), aiming to reach all teachers of mathematics in primary, secondary, and adult education in Sweden through a one-year professional development program.

The case study teacher works in a context where language has the potential to be a powerful barrier to meaningful learning, making the constraints of the context on the implemented curriculum particularly visible. The teacher uses this as a starting point for planning engaging lessons that identify learners' starting points, probe meaning-making in depth, and build up the language skills required for reasoning, applying knowledge, and problem-solving. At the same time, she supports the development of shared

communication and positive attitudes that further support mathematical meaning-making and problem-solving as learners mature.

School, Subject, and Teacher Context

Sara Svanlund has completed five years of university education. She teaches Grades 7–9 (ages 14–16) at Möllevångsskolan, a school in the center of Malmö for learners aged 6 to 16. Her learners come from very mixed family and socioeconomic backgrounds. They have 240 minutes of mathematics instruction per week, divided into four lessons, and she is frequently supported in the classroom by a study supervisor or a mother tongue teacher.

Sara explains that many learners at her school do not have Swedish as their mother tongue; consequently, there has been a strong school-wide focus on language development, including within mathematics. Teachers in her school have found that a shared proficiency in the Swedish language is key to understanding and solving mathematical problems, as many concepts are difficult to access without the ability to read and write the language used to express and interpret mathematical ideas. Unfortunately, too many learners do not achieve passing grades when they leave this junior high (lower secondary) school. Teachers therefore focus on both language development and foundational mathematical understanding.

For example, Sara might begin a lesson with an incorrectly solved task. The class discusses why and where the solution went wrong and how they might fix it. During this process, she pays close attention to challenging words and concepts, helping learners develop the language required to participate confidently in mathematical reasoning.

More generally, Sara tries to integrate language development into practical mathematics tasks by using examples from learners' everyday lives. Her lessons use varied groupings (for example, groups, pairs, and individual activities) to sustain engagement. She believes it is important to identify learners' starting points and build instruction from there; the questions teachers ask and the responses learners provide are key to monitoring understanding. By analyzing incorrect or unexpected answers, she gains insight into learners' thinking and can determine appropriate next steps in instruction and learning. For Sara, practical and fun mathematics is important. She finds that many of her learners label mathematics "hard" or "boring," so she plays a lot of mathematics games and competitions in her lessons, resulting in her learners making good progress and enjoying their mathematics lessons.

**Didactic principles in Sara's classroom:**

- **Lessons maintain a dual focus on language development and deep understanding of fundamental mathematics.**
- **Instruction begins from learners' existing knowledge and builds from there, with the teacher questioning and learner responses serving as key tools for monitoring understanding.**
- **Varied learner groupings and activities, including practical tasks from their everyday lives, ensure continued learner interest and support participation.**

3.6 Case Study 5: Shifting Toward More Learner-Focused Approaches in Malta in Grades 8–10

TIMSS-related developments in Malta

The National Curriculum Framework, which guides compulsory schooling in Malta, was launched in 2012 in response to, among other influences, learners' performance in TIMSS 2007. The National Curriculum Framework aims to address gaps in Malta's learning processes identified through TIMSS by shifting the emphasis of classroom instruction from teaching the subject to teaching the learner. It also aims to support greater equity and decentralization of the national education system while emphasizing smooth educational transitions and building on the strong foundations established in early childhood education.

The accompanying Learning Outcomes Framework gives schools the freedom to develop programs that fulfill the objectives of knowledge, attitudes, and skills-based outcomes to which all learners in Malta are entitled. The Learning Outcomes Framework is intended to increase curricular autonomy for colleges and schools, allowing them to respond more effectively to the needs of their learners.

The experienced teacher in this case study applies these principles to offer his learners regular opportunities to solve genuine problems. These tasks typically require the use of a range of content and cognitive domains from the TIMSS framework, as they support deeper meaning-making around core concepts and the application of mathematical reasoning in unfamiliar contexts. He aims to develop a positive and collaborative classroom environment in which learners come to see mathematical challenge and struggle as productive experiences that, when coupled with persistence and teacher and peer support, are likely to result in confident and successful mathematical engagement with the world.

School, Subject, and Teacher Context

Alexander Ellul teaches at St. Benedict College Secondary School, an all-attainment state school in the village of Kirkop, southern Malta. It caters for approximately 750 learners in Grades 8–10 (ages 13–16). The language of instruction is English, although that is not the first language of many learners. Alexander has been teaching mathematics for 26 years. He has a Bachelor of Education in Mathematics and a Master of Education in Research in Education. Each year he teaches two classes from Grades 8–10. The prior attainment levels of those classes vary each year, but each class receives five 40-minute mathematics lessons each week.

Alexander's account shows how the curriculum changes in Malta have impacted individual teacher practice. He says that when he started teaching, his philosophy was to make the subject as easy as possible so that learners would be able to perform better. He wanted every learner in his class, particularly those who were struggling, to understand the basic idea of the concepts being discussed by starting with simple examples and gradually progressing to harder ones. This was achieved through direct instruction and a whole-class approach focused on how learners should think and how they should proceed. His approach has evolved in light of curriculum and assessment reforms triggered in part by learners' performance in TIMSS. He now seeks to provide an environment where learners can think for themselves about how to approach a given situation by asking meaningful, indirect, and provocative questions to support their mathematical reasoning.

In parallel, his approach to assessment of learning has also changed to mirror the national move toward more continuous assessment. Learners receive credit for their year-round achievement through the assessment of tasks that differ from those used for summative assessments (for example, examinations). This approach provides a more complete and balanced picture of their learning journey, rather than relying solely on the end-of-year exam results.

Alexander has therefore started using more non-routine tasks for continuous assessment in order to support greater meaning-making and critical thinking. By exploring, discussing, and reflecting on the task at hand, the role of memorization of facts and procedures is diminished and the focus on reasoning enhanced. The tasks are designed to move learners beyond standard procedures, requiring them to apply their understanding in unfamiliar contexts. Initially, of course, not all learners feel comfortable working in this way. However, the intention is to help them develop genuine problem-solving skills. Rather than giving step-by-step instructions, Alexander encourages learners to reason things out—sometimes individually and sometimes in pairs. He also expects that they justify their thinking and are able to communicate that to others. He

finds that this approach helps learners gain confidence in their mathematical abilities and develop a more creative mindset.

In such tasks, what is important to Alexander is that learners are not simply following procedures but that they are making sense of what they're doing. He praises them for presenting thoughts and observations, even when they make mistakes, as these ideas can later be refined through further reflection. He emphasizes that we can have a range of different, but still valid, responses to the same question.

As a result, Alexander says his teaching has become more focused on the learning process rather than just the outcome. With effort, he has developed the ability to allow learners to struggle in meaningful ways. He notes that sometimes the silence following a posed question or situation can be highly productive and should be valued, as it provides learners time to think.

Alexander says the recent assessment reforms have made him more reflective in his teaching, allowing him to continuously adapt his support to different learners and to focus on developing mathematical skills in a curious and interesting manner.

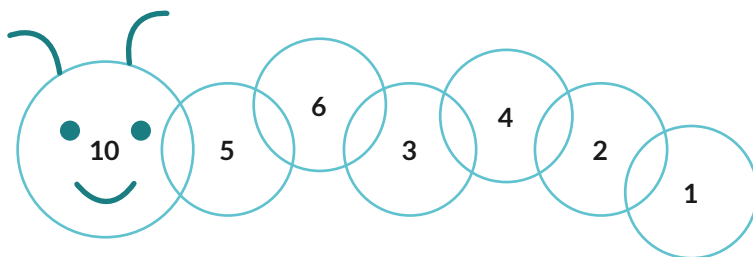
To illustrate his approach, Alexander gives the example of "The caterpillar problem." This task ([Figure 3.3](#)) is designed for Grade 8 learners (although it is accessible to a wider age range) and is based on a simple mathematical rule: if the number is even, halve it; if it is odd, add one. The number on the caterpillar's head represents its age, and learners must figure out how many body parts the caterpillar has by applying the rule repeatedly until the last part has number 1.

Figure 3.3 The caterpillar problem

A caterpillar's age is written on the head. Each body part is worked out from the previous one in the following pattern:

- If the previous number is even, halve it.
- If the previous number is odd, add one.

The pattern continues until you reach 1 on the final part. For example, a caterpillar of age 10 has a head and six body parts:



- 1)
 - a) How many bodyparts does a caterpillar of age **12** have?
 - b) How many bodyparts does a caterpillar of age **11** have?
 - c) How many bodyparts does a caterpillar of age **50** have?
 - d) How many bodyparts does a caterpillar of age **49** have?
- 2) Caterpillars do not live beyond 100 years old. What is the age of the longest caterpillar? How do you know?
- 3) Suppose there is no age limit for a caterpillar. How long a caterpillar can you find now?

Learners work in pairs so they can explain their reasoning to one another and explore different possibilities together. This creates a supportive environment where learners can test their ideas—and also supports mathematical communication. Alexander does not respond directly to queries but guides learners back to the task (the language is deliberately straightforward since English is not the first language of many of his learners). He encourages them to check steps carefully, looking for shortcuts or patterns, and comparing strategies. Some learners write down every step manually, while others begin to notice that certain ages lead to repeated patterns.

The second question, "What is the age of the longest caterpillar?", is particularly challenging. Many learners do not arrive at the correct answer; however, Alexander is more interested in the mathematical processes they use in working toward a solution. He focuses on how learners record their steps, whether they notice patterns, whether they find ways to be more efficient, and how they justify their conclusions. Overall, he values mathematical thinking rather than correct answers.

Didactic principles in Alexander's classroom:

- **Space is created for mathematical exploration.**
- **Emphasis is placed on the process rather than correct answers.**
- **Reasoning skills are developed beyond procedural recall.**
- **Instruction prioritizes mathematical reasoning and competence, supporting learners to engage meaningfully with mathematics.**



3.7 Case Study 6: Embracing Higher-Order Thinking Skills in Malaysia in Grades 10 and 11

TIMSS-related developments in Malaysia

Malaysia has participated in every cycle of TIMSS since 1999, with performance generating a lot of interest among academics, politicians, stakeholders, and even the public. This has raised awareness among Malaysians of the importance of upgrading the quality of mathematics and science learning, in addition to reviewing the curriculum. Among the measures taken have been the training of teachers to integrate Higher-Order Thinking Skills objectives into their classroom instruction, reviewing textbooks, and increasing the proportion of Higher-Order Thinking Skills items in national examinations. A revised mathematics and science curriculum at the primary and secondary levels was introduced in 2017, beginning with Form 1 and Year 1, respectively.

This case study again underlines the centrality of the context for framing the implemented curriculum. The teacher works with classes coming from diverse social and mathematical backgrounds within a system where historical approaches have been relatively closed—focusing on facts and algorithms rather than meaning-making. However, he encourages meaning-making through approaches that reach beyond core knowledge. These approaches are collaborative and focused on problem-solving, grounded in shared mathematical reasoning. He argues that such approaches also support positive attitudes and the mathematical confidence necessary for ongoing mathematical participation.

School, Subject, and Teacher Context

Muhammad Fazdhly bin Abdul Muttalib teaches at Sekolah Menengah Kebangsaan Raja Permaisuri Bainun in Ipoh, Perak Darul Ridzuan, Malaysia. This is a mixed state-supervised school located in an urban area. Admission is based on learners' performance in primary school (for entry to Form 1, typically at age 13) and in lower secondary school (for entry to Form 3, typically at age 15). The school attracts learners from a range of socioeconomic backgrounds. All learners board full-time in the school hostel, coming from all over the state of Perak. Muhammad teaches additional mathematics to Form 4 (Grade 10) learners (ages 16–17) and mathematics to Form 5 (Grade 11) learners (ages 17–18). Each subject is allocated three hours a week (six 30-minute teaching periods).

Muhammad uses a variety of teaching approaches to support learning mathematics with meaning, adapting his strategies to the needs of the class. These include:

Direct instruction: Learners are directly exposed to ideas, concepts, mathematical formulae, and example questions along with their solutions. Muhammad describes this approach as time-efficient for addressing core content related to facts and procedures. However, he does not use it frequently, as he believes that for most learning purposes, learners need to be more actively involved.

Inquiry-based teaching: Muhammad believes this is effective in helping learners grasp mathematical concepts clearly and confidently, via both confirmatory inquiry and structured inquiry. His experience suggests that mathematics instruction through inquiry helps learners understand concepts more clearly because they are actively involved in the learning process. This approach aligns with the Ministry of Education Malaysia's emphasis on meaningful learning by making learners active learners.

Twenty-first century learning: This approach emphasizes collaboration among learners during the learning process. Muhammad's experience is that mathematics teaching that uses 21st-century learning tools encourages learners to be actively involved in the teaching and learning process, developing a deep curiosity about a particular mathematical concept and resulting in them retaining ideas and information for the long term.

Cultivating Higher-Order Thinking Skills: Muhammad believes that the use of diverse teaching approaches can sharpen learners' critical and creative thinking skills. When learners are constantly engaged in thinking and striving to solve mathematical problems—supported by high-quality teacher questioning—their Higher-Order Thinking Skills develop naturally. Furthermore, they become increasingly prepared to reason logically around the given problems and the solutions obtained.

Muhammad gives an example, for Form 5 (Grade 11), when mathematics learners are given the opportunity to explore the effects of transformations (for example, translation, reflection, rotation, and enlargement) on geometric shapes.

In this example, he prepares eight stations, each consisting of transformation instructions for the learners. Learners are divided into eight groups, each of which has a two-dimensional geometric shape drawn on the Cartesian plane. Groups move from station to station to perform the required transformations and draw the resulting images. They discuss approaches to solve the problems, checking their answers via answer keys provided at each station. The teacher acts as a facilitator to assist, support, and challenge learners with questions, uncertainties, or probes to stimulate thinking.

After a complete rotation, a representative from each group presents their results to the whole class and shares the group's experiences. They will also highlight any mistakes made and explain how they overcame them, since significant learning often occurs at such moments.



Didactic principles in Muhammad's classroom:

- **A combination of instructional approaches is used over time** to support mathematical meaning-making, including direct instruction, inquiry-based teaching, and the specific targeting of transferable twenty-first century skills such as collaboration.
- **Higher-Order Thinking Skills are explicitly targeted**, including critical and creative thinking.
- **Whole-class, small group, and teacher-learner discussions are used to expose and address emerging thinking**, including misconceptions as well as elements of mathematical reasoning.

In Summary

- **Global use of TIMSS findings:** Findings from TIMSS have been used in multiple ways across the globe to interrogate both national and comparative practice. The findings have informed a variety of changes in mathematics policy across curriculum, assessment, promoted pedagogies, teacher education, and centrally provided curriculum resources.
- **Building across cognitive and content domains:** Mathematical meaning-making and authentic problem-solving are often underpinned by building across cognitive and content domains.
- **Deliberate development of positive attitudes to challenge:** Teachers need to intentionally build positive attitudes toward challenge, "getting stuck," and persistence if learners are to deepen their understanding of core ideas and apply them effectively in reasoning and problem-solving contexts.
- **Learning from cross-cultural differences:** Because teaching and learning practices are deeply embedded in a country's cultures and local contexts, they do not always transfer unproblematically to another situation. However, differences in practice can be used to ask questions and offer ideas for further exploration.
- **TIMSS-catalyzed changes are impacting the classroom practice of teachers across the globe:** TIMSS insights very often result in ways that further promote the learning of mathematics with both meaning and positive attitudes.

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Source

Golding, J., Marks, R. (2026). Responses to TIMSS That Support Meaning-Making: Inspiring Examples From Around the World. In: Teaching and Learning Mathematics with Meaning. *IEA Research for Educators*, vol 4. Springer, Cham.

https://doi.org/10.1007/978-3-032-35352-8_3

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