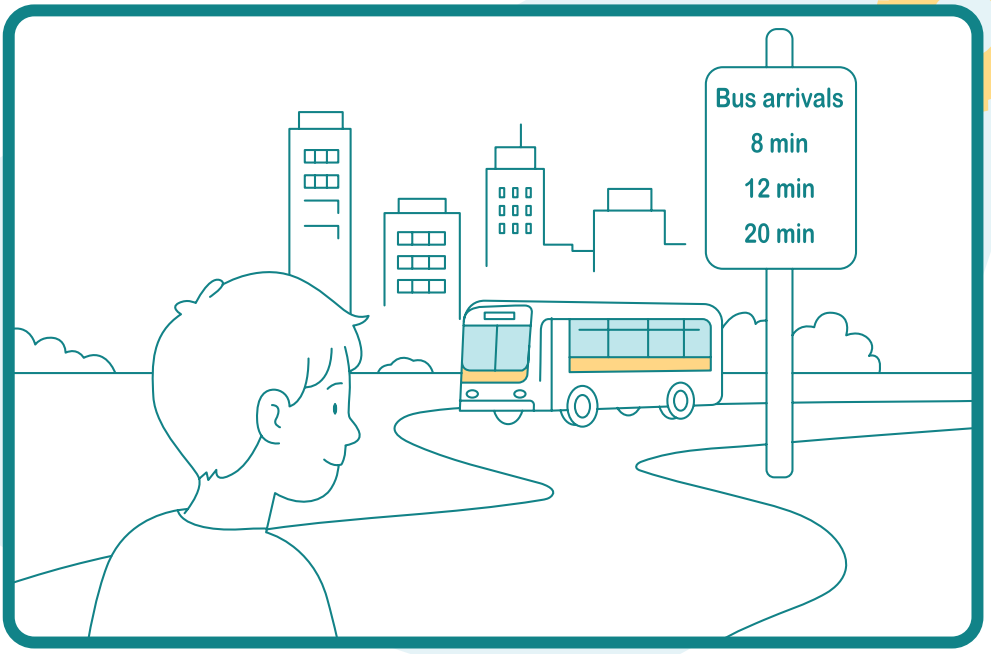
Teaching and Learning Mathematics with Meaning: Using and Applying TIMSS

Classroom Materials Grades 3-6

10 TIMSS items with didactic notes



Content Domains:
Data, Measurement and Geometry, and Number

Cognitive Domains:
Applying, Knowing, and Reasoning

TIMSS Benchmarks
Low, Medium, High, and Advanced

Contents

Classroom materials for Grades 3-6

These materials consist of selected Trends in International Mathematics and Science Study (TIMSS) items, together with accompanying didactic notes to support their use in the classroom. These items were designed for Grade 4 learners participating in TIMSS assessments, but they are equally suitable for a range of other grades, depending on how the teacher uses them.

Each TIMSS item is presented on a separate page with descriptive nformation, with answers available in a separate sheet at the end. This information helps teachers identify items that align with their curriculum, learning objectives, or instructional focus. The descriptive information is as follows:

Content Domain:
TIMSS assesses Grade 4 mathematics across three content domains: *number*, *measurement and geometry*, and *data*. Together, these domains represent the key areas of the primary mathematics curriculum and measure students' understanding and application of fundamental mathematical concepts.

Cognitive Domain:
TIMSS assesses Grade 4 mathematics across three cognitive domains: *knowing*, *applying*, and *reasoning*. Together, these domains represent the cognitive processes that students develop as they learn mathematics, measuring their ability to understand concepts, apply knowledge, and reason effectively in a range of mathematical situations.

TIMSS Benchmark:
TIMSS reports student achievement at four international benchmarks: *Low*, *Intermediate*, *High*, and *Advanced*. These benchmarks describe increasing levels of mathematical knowledge and understanding.

Description:
The description of the task explains what students are asked to do and highlights the mathematical concepts and cognitive processes the task is designed to assess.

The accompanying **didactic notes** appear on the page following each item. They explain why the item is important and what makes it challenging and suggest practical teaching approaches and ideas for using the item in classroom practice.

Chapter 4 of *Teaching and Learning Mathematics with Meaning: Using and Applying TIMSS* (Golding and Marks; 2026) illustrates how a teacher might use one or more of these items in the classroom. However, a variety of other approaches are possible, and teachers will rightly choose to use them differently at different stages of learners'

growing mathematical knowledge and confidence. What is important is that learners' mathematical *thinking* and *meaning-making* are focused on, rather than primarily the "correctness" of answers and the procedures used to obtain those answers.

The items, together with the percentage of students who answered each question correctly, may be found and downloaded for individual classroom use from the IEA TIMSS & PIRLS Website[1]. We suggest teachers work through the items themselves to understand the mathematical (and other, such as reading) demands involved.

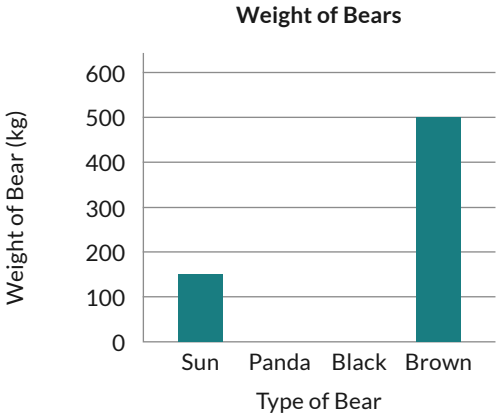
[1] TIMSS 2019 International Results in Mathematics and Science (https://timss2019.org/reports/achievement/index.html). See Mathematics Grade 4 example items in the Performance at TIMSS International Benchmarks in Mathematics.

1. **Content Domain**: Data
   Cognitive Domain: Applying
   TIMSS Benchmark: Low
   Description: Represents data from a table in a bar graph

The table shows the weights of 4 bears.

Type of Bear	Weight (kg)
Sun	150
Panda	200
Black	250
Brown	500

Use the data to complete the graph.



See answer on page 37

Why is it important? Bar graphs are a foundational tool that support a wide range of cognitive and practical skills in young learners. Learners develop skills in interpreting and analyzing data visually and in communicating information clearly and effectively. Such skills are important given the growing datafication of the world and will be valuable across subject disciplines and in later life. Further, in such a context, learners begin to compare quantities, identify trends, and make inferences, providing a strong foundation in critical thinking.

How difficult is this task? On average, 81% of Grade 4 TIMSS learners were able to complete this item successfully.[2]

What makes it difficult? Younger learners may find interpreting and completing bar graphs, as required in this question, difficult for several reasons. Understanding and reading the scales on the axes can be tricky. Learners need to apply a fluent and flexible understanding of number to interpret the intervals and must be able to read the height of bars that fall between two labels on the y-axis. They may have previously encountered graphs with different scales, so teachers need to ensure learners are not overgeneralizing from previous experiences. Drawing the missing bars requires attention to detail, with learners needing to apply visual-spatial skills when judging the lengths of the bars. Particular attention should be paid to learners with weaker fine motor skills who may benefit from alternative ways to demonstrate their understanding. Language may also present a barrier, with some learners—particularly EAL (English as an Additional Language) learners—not recognizing "sun," "black" or "brown" as types of bear and therefore not appreciating the comparative context.

Didactic approaches:

- Begin, briefly, with a set of small model or toy bears. Ask how they might be grouped and physically sort them into separate hoops. Then ask whether this approach can be applied to real bears. Elicit learners' knowledge about different types of bears and discuss how they might be compared or grouped.
- Show images of the four bears used in this question, scaled to reflect the data (i.e., ensuring the brown bear is the largest image). Ask learners how they might compare the bears. Tabulate different characteristics (color, height, how many sandwiches they have for lunch, etc.) and support learners in understanding how to read the table.
- Show learners a similar table to the one used in the question but with data for "How many sandwiches did this bear eat for lunch?" (sun = 5; panda = 0 [they only eat bamboo]; black = 20; brown = 15; Paddington = 45 [all marmalade, as Paddington Bear only eats marmalade sandwiches]). Ensure learners are comfortable reading the table. Ask for ideas about how else the information might be displayed. If

[2] Countries' Mathematics and Science Achievement – TIMSS 2019 International Reports: https://timss2019.org/reports/achievement/index.html#math-4

appropriate, provide examples of bar graphs drawn from media familiar to learners.

- Work with learners to construct a class bar graph of the "sandwiches" data, starting with the sun bear. Use a 1:1 scale with a y-axis of 30 squares in height. Pause at Paddington and ask what is problematic. The dramatic addition of a further piece of paper to extend the y-axis can help to emphasize the importance of considering scale.
- Discuss what could be done to make the graph fit on one page. Remind learners that counting in groups or jumps can make counting easier. For example, learners might "race" to count to 100, with one group counting in 1s and another in 10s. Show that the bar graph becomes easier to draw and understand when a grouped scale is used. Ask what a sensible grouping might be in this case. Practice counting in different groups and explain that counting in 5s aligns with each data point.
- Redraw the graph with the y-axis marked in 5s. Look at how the data now fit and how the graph is easier to read. Check comprehension by asking data comparison questions such as, "How many more sandwiches did Paddington eat than the brown bear?" Then pose a further question: "What would happen if I was feeling a bit lazy and hadn't filled in all the numbers?" As you say this, erase 5, 15, 25, 35, and 45 from the y-axis. Explore with learners how they can still determine what the missing numbers on the gaps would be, even without the numbers being there. Take this further with some number lines (horizontal and vertical) showing regular intervals and ask learners to identify missing points.
- Now show learners the table from the original question with the bears' weights. There is a lovely opportunity here for learners to look up and add the weight of Paddington Bear (an Andean bear, approx. 150 kg[3]). Ask learners what they notice from the table. Discuss order and comparison, using the original bear images for sorting if helpful.
- Explain that you have already started a bar graph using this information and show the graph from the question. Ask learners what they notice about the y-axis. Practice counting in 50s. Explore how the data in the table match the partially completed graph. Now invite learners (on their own devices, in pairs if possible) to draw in the bar for the panda. Check class agreement and address any misconceptions. Then examine the data for the black bear. The weight is 250 kg, but this is not on the scale—what could be done? Ask learners to discuss in pairs and complete the graph—again, address any misconceptions.
- Dependent on learner needs, provide another dataset for learners to complete a similar exercise (this time, you might want them to draw the axis themselves). Alternatively, they might research an area of interest and then produce their own graph from the data. Practicing explaining their graph and its data will also aid their mathematical communication skills.

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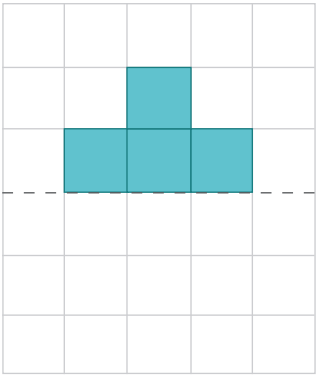
[3] See, for example, https://www.bbcearth.com/news/the-real-life-paddington-bear

2. Content Domain: Measurement and Geometry
Cognitive Domain: Applying
TIMSS Benchmark: Intermediate
Description: Completes a symmetric figure on a square grid given half the shape and the line of symmetry

Complete this figure so the dashed line is a line of symmetry. Colour in complete squares.



See answer on page 37

Why is it important? Symmetry is a versatile concept that supports a wide range of cognitive, creative, and practical skills in young learners. It helps learners understand geometry, shapes, and spatial relationships, which are essential for more advanced mathematical topics. Recognizing and working with symmetry enhances visual perception skills, enabling learners to notice patterns, organize visual information, and improve their ability to observe and describe objects. When working with tasks such as this, learners develop new vocabulary and improve their ability to communicate mathematical and spatial concepts.

How difficult is this task? On average, 70% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? Symmetry involves understanding abstract ideas about balance and reflection, which can be challenging for young learners to grasp. It requires strong visual-spatial skills to recognize and create symmetrical patterns, as well as careful attention to detail to ensure that both sides of a shape or pattern are identical. This can be difficult for those who are still developing fine motor skills. With respect to the format of this question, many learners—particularly from their earlier kindergarten and school experiences—will be familiar with vertical mirror lines (for example, when folding paper to create butterflies from paint). They need to understand that mirror lines can occur in any orientation.

Didactic approaches:

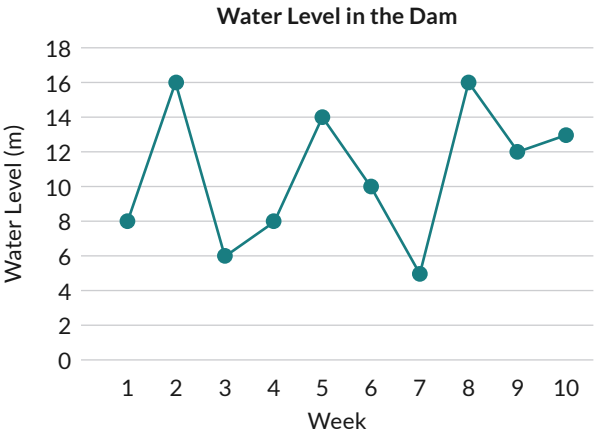
- Practical experiences will support an embodied understanding of the concept of a mirror line. Use masking tape to create a mirror line in the classroom or playground. Learners should work in pairs, with those on one side of the line being the reflection. Ask one learner to perform a range of movements: walking toward the line, walking away from the line, raising their left hand, standing on their right foot—what does the learner mirroring their actions do? Careful use of language is needed to emphasize the idea of mirroring, not copying.
- Use digital tools to further develop the idea of a mirror line. Display an image of half a face (sliced vertically)—what happens if it is mirrored? Try this with different images, all with a vertical mirror line.
- Replace the image with the top of a head and now introduce a horizontal mirror line—what would happen? Use digital tools to model the outcome. Discuss the distance of parts of the image from the mirror line—for example, "My glasses are 5 cm from the mirror line in the original image and in the reflection."
- Where do learners encounter reflections? If you have access to a pond, river, or even puddles, you might go outside to observe and photograph reflections and symmetry to create a display.
- Put a horizontal mirror line on the board and ask pairs to copy it onto their mini whiteboards. Add a single solid-colored square above, touching the mirror line. Ask

learners to discuss with a partner where the reflection would go and to draw this on their boards. Now move the square one square's distance from the line. How should the reflection be drawn now? Try a range of positions until all learners are comfortable, then introduce multiple blocks.

- Learners might play a symmetry game involving a physical line and using multi-link cubes. One learner creates a pattern while their partner produces the reflection.
- Use the example question on the board. At this stage, ensure learners understand what the question is asking and are confident in using the technology. Learners might then work through a series of increasingly complex examples. Several apps focus on symmetry and can provide learners with interactive practice and immediate feedback.

3. Content Domain: Data
Cognitive Domain: Knowing
TIMSS Benchmark: Intermediate
Description: Reads data from a line graph

The graph shows the water level in a dam for 10 weeks.



What was the water level for week 8?

Answer: _____ m

See answer on page 37

Why is it important? In an increasingly data-driven world, it is important that learners develop the necessary data literacy to support them in their adult lives. They need to understand different types of graphs, when they are used, and how to read them.

How difficult is this task? On average, 68% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? It is likely that Grade 4 learners will have had limited exposure to line graphs and therefore may have little experience reading or constructing them. There are several elements learners need to understand to be successful in this item. They must comprehend that a line graph displays trends over time and understand what the points and lines represent. They need to recognize that both the x-axis and the y-axis contain important, related information and learn to coordinate information between both axes simultaneously. Further, each axis has a different scale, so learners must understand intervals and hold information from two scales simultaneously. Processing these multiple elements—particularly if the underpinning skills are weak—may impose a high cognitive load on learners, making the task more difficult.

Didactic approaches:

- Approaches will depend on learners' previous experience with line graphs (or other graphs). If they have yet to encounter line graphs, it may be better to use a cross-curricular approach and introduce them within a meaningful context, for example, in science or geography. Learners might, for example, work as a class to produce a temperature chart (when all temperatures are positive), where they will receive daily reinforcement on how to plot points on the chart. If there is data available on a variable such as the water level near to the school (and therefore meaningful to the learners), such data could be used to construct a graph over time. Attention should be drawn to intermediate time points so that the graph not only shows the data for a particular point but also gives "in-between" data, such as halfway between weeks 1 and 2.
- Look at a class version of the graph above (or one with similar data on a week-by-week basis in a context familiar to the class). Focus on the x-axis. What does it show? Why is every number included? What does the space between the numbers represent?
- Look at the y-axis. What do learners notice about the differences between this and the x-axis? Draw attention to the interval of 2. Why might this interval have been chosen (for example, to save space with a dataset that has a wider range)? Look at the data points for weeks 1–6 and read across to the y-axis (all of which are multiples of 2). Most learners will be competent in reading these data points. Then examine the data point for week 7. Why is it positioned between two horizontal gridlines (4 and 6)? How should it be read? **Note that this is beyond the demands of the TIMSS item but an important skill for learners of this age to develop.**
- Ask learners to find a range of line graphs in the media and create a class collection of suitable examples (y-axis intervals of 1, 2, 5, or 10). Practice reading data points.

- Learners might collect or be given data from which to construct their own line graph. Stress the importance of using an appropriate scale and the use of straight lines to join data points. Note the continuous nature of the variable represented on the x-axis and the ability to therefore read between data points.

4. Content Domain: Number
Cognitive Domain: Applying
TIMSS Benchmark: High
Description: Identifies an expression that represents a situation

There were 12 liters of water in the tank.

Ravi then pured 3 liters of water into the tank and Indira pured another 3 liters of water into the tank.

How can the amount of water in the tank be calculated?

Ⓐ 12+(2+3)

Ⓑ (12+3)+(12+3)

Ⓒ (12+2)x3

Ⓓ 12+(2x3)

See answer on page 37

Why is it important? Retelling an everyday situation using mathematical terminology supports learners in beginning to use ideas of mathematical modeling. Mathematical models help us solve complex problems by providing a structured way to analyze and understand real-world situations. They allow us to break down problems into manageable parts and find solutions. As learners progress through their education, modeling will underpin much of their mathematical experience. Beyond the experience of moving between the real and mathematical worlds, questions such as this require learners to have a fluent and flexible understanding of the concepts of both addition and multiplication and the ability to translate between additive and multiplicative structures; such skills are fundamental to learners' success when they encounter algebra in high school.

How difficult is this task? On average, 53% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? Learners must grasp the context of a real-life situation and identify the relevant information that can be modeled mathematically. This requires strong comprehension skills and the ability to filter out irrelevant details. As this problem involves multiple steps, learners need to break down a problem into manageable parts and determine the appropriate mathematical operations to use. Many learners will want to revert to the sole use of addition (12 + 3 + 3). Weak mathematical flexibility will make it difficult for some learners to see that 3 + 3 can also be represented as 2 x 3 here. Learners will also require a basic understanding of brackets, which may be new to them, to comprehend the possible answers to this task.

Didactic approaches:

- To support learners in beginning to engage with ideas such as this, use relatable and engaging examples, provide plenty of practice, and break down the process into smaller, manageable steps. Model the situation physically with a tank of water containing 12 liters. Add 3 liters and ask learners to work in pairs to identify the calculation needed. Emphasize the interest in the calculation, not the answer.
- Repeat the above using the same situation (water being poured into a tank) with a range of other one-step problems:
    - There are 10 liters of water in the tank, and Peter adds 5.
    - There are 4 liters of water in the tank, and Xavier adds another 4.
    - The tank is empty. Stef adds 6 liters and James adds 2 liters.
    - There are 8 liters of water in the tank, and Charlie takes out 3 liters.
    - The tank is empty. Katie adds two full 5-liter buckets of water.
- In each case, emphasize that there is no question—we are interested in the calculation. If necessary, ensure learners are aware of extraneous detail (the names of the children do not matter, for example).
- Use the example in the question, but first model it physically again. Do not show the options; instead, allow learners to come up with their own responses. It is likely

that they will suggest 12 + 3 + 3. Be careful to ensure learners understand that this is correct and that we are interested in different ways of representing the same calculation before moving on. Discuss (and remodel if needed) the actions, explicitly using the language: "Two lots of 3 were added." Ask learners (pair discussion): "How do we write 'two lots of three' mathematically?" Once learners have established that 3 + 3 can also be written as 2 x 3, rewrite the calculation 12 + 3 + 3 as 12 + (2 x 3). If necessary, introduce brackets in a light-touch way.

- Repeat the above with another similar example.
- Repeat again, this time using "big numbers" to emphasize the mathematizing element rather than the calculation. For example:
    - There are 112,000 liters of water in the swimming pool. On Monday, Sarah adds 547 liters. On Tuesday she adds another 547 liters. How can the total amount of water in the swimming pool be calculated?
- To assess learners' understanding of the process, ask them (in pairs) to create their own story problem and four possible calculations (only one of which should be correct), with partners solving each other's stories. To deepen learners' understanding, they might be presented with some multistep calculations and be asked to write a matching story.

5. **Content Domain**: Number
   Cognitive Domain: Applying
   TIMSS Benchmark: High
   Description: Solves a word problem involving subtraction of a non-unit fraction from 1

Anna is cycling to her grandmother. She has cycled $\frac{3}{8}$ of the way.

What fraction of the distance does Anna have left to cycle?

Answer: $\frac{\square}{\square}$

See answer on page 38

Why is it important? Fractions are fundamental to many aspects of mathematics and everyday reasoning. They form the foundation for later areas of mathematics; therefore, developing a strong grounding early on gives learners the fluency and flexibility required to tackle more advanced mathematics. Questions like this underpin everyday reasoning and relate to problems of ratio. Being confident in the processes and procedures involved will support learners' broader mathematical confidence.

How difficult is this task? On average, 47% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? Many primary school-aged learners struggle with computation of fractions due to a weak conceptual understanding of what fractions are and how we operate with them. To be successful in this question, learners first need to know that the whole journey is 1, which is equivalent to $\frac{8}{8}$. This may be difficult as the whole is not given in the question; learners are required to infer it from the wording. Learners then need to be able to manipulate fractions to calculate 1 - $\frac{3}{8}$. At this age, many learners may try to apply standard procedures without a full understanding of how they apply to fractions. The most successful learners will be those who can construct a physical, pictorial, or mental model and understand that they are looking for the fraction that needs to be added to $\frac{3}{8}$ to make $\frac{3}{8}$.

Didactic approaches:

- It is important for an item such as this to be presented only once learners have developed the necessary foundational conceptual understanding of the underpinning components. They need to understand what fraction notation represents in order to visualize fractions less than 1, develop a sense of their magnitude, and recognize that the "whole" (the total journey) can be represented as 1.
- In looking at this item specifically, start with a horizontal linear empty number line. Mark "home" and "grandmother's house." Perhaps have a picture of a bicycle (or something to represent it that can be moved along the line). Place this at "home" and ask learners how much of the journey has been completed. Elicit that this is 0, the starting point, and mark 0 on the number line. Then look at "grandmother's house" and establish that the whole journey is complete at that point. Ask learners how we can represent this on a number line when we do not know the actual distance—what do we use to represent a whole (1)?
- Now introduce the information that she has cycled $\frac{3}{8}$ of the journey. Ask where this would be placed on the line and how they know. This will depend on learners' prior experiences but may involve dividing the line into eighths (or possibly marking half and quarter as benchmarks). At this stage, depending on the models learners are familiar with, it may be helpful to translate the information into a bar model so that learners can more easily visualize the part they are trying to find. Support learners to find the remaining block, dividing into eighths if necessary.

- Mental and flexible approaches should be prioritized in this item. It would not be appropriate for learners to attempt subtraction of fractions using a formalized approach and while it would be appropriate to draw their attention to the answer being 1 - $\frac{3}{8}$, this should not be calculated formally.

6. Content Domain: Data
Cognitive Domain: Reasoning
TIMSS Benchmark: High
Description: Represents data from a table in a pictograph

Animal Weights

Animal	Weight (kg)
Cheetah	50
Lion	100
Leopard	75

Make a pictogram of the weight of each animal.

Use the symbol  and  to complete the pictogram. The cheetah has been done for you.

Animal	Weight (kg)
Cheetah	
Lion	
Leopard	

Key  =50 kg

See answer on page 38

Why is it important? Working with pictograms builds aspects of learners' data literacy in an accessible and meaningful context. Data literacy skills are increasingly important in our data-driven world and will allow learners to navigate and critique data and data presentations with confidence. Working with pictograms supports key mathematical concepts such as counting and grouping, comparing quantities, and understanding scales and keys, all of which are foundational for later work in statistics and probability. Creating pictograms encourages learners to think about how best to communicate information clearly. It also provides opportunities for learners to explain their reasoning and justify their choices, both important aspects of mathematical communication.

How difficult is this task? On average, 61% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? When learners have previously encountered pictograms, they are likely to have used images that represent the item in question—here, they would have drawn cheetahs, lions, and leopards. However, this question moves from "how many" to a property of the animals, and learners need to shift attention to the weight of the animal, not to the animal itself. Further, they need to understand that the image used in this question stands for 50 kg (they may well have seen it representing a different mass in another task). They also need to have the mathematical flexibility to work out that if the whole is equal to 50 kg, half would be equal to 25 kg. While no single aspect of the process is complex, there are multiple points at which learners might make mistakes or hold misconceptions.

Didactic approaches:

- This item moves learners beyond earlier pictogram work they would have encountered to introduce symbolic notation, variables, and keys. Begin by recapping what a simple pictogram is and how it is read. Construct one using the three animals listed, with each image representing one animal. Ensure the images used have vertical symmetry. Explain that these represent a record of the number of animals in a zoo, and reinforce that counting allows us to determine how many of each there are.
- Look at a pictogram for animals in a safari park with a key showing that one image represents 10 animals. Explain that with larger datasets we need to use one image to represent multiple animals, as there are too many to draw. Show rows for cheetahs (40 represented by 4 symbols) and lions (70 represented by 7 symbols) and ask learners to work out how many of each there are using the key. Then explain that there are 35 tigers. Ask how this could be represented in the pictogram. Partition 35 into 30 and 5. Represent the 30 with 3 symbols. Then ask how the remaining 5 should be shown. Elicit from learners that if one symbol represents 10 animals, half a symbol must represent 5. Add the half symbol to the pictogram.
- Move to an example of the amount of food eaten by each animal with a key showing that one bag represents 50 bags of animal feed. Emphasize that the image is the

same now for each animal, as we are representing what they eat, not the animals themselves. Provide data similar to that in the weight table and ask how we would show 75 bags of food.

- Look at the TIMSS item—this has been started but not completed. Re-emphasize features such as the use of the same symbol and the key. Examine the numbers in the table—are they all multiples of 50? If not, what will we need to do to represent the other numbers? Complete the pictogram for lions and tigers.
- Learners should practice with a range of examples, converting data tables into pictograms and vice versa.

7. Content Domain: Number
Cognitive Domain: Reasoning
TIMSS Benchmark: Advanced
Description: Devises two ways of grouping objects that satisfy two conditions

A teacher wants to put 30 children in groups so that:

- each group has the same number of children, and
- each group has an odd number of children

Find two different ways the teacher could make the groups.

Way 1

Number of groups: ☐
Number of children in each group: ☐

Way 2

Number of groups: ☐
Number of children in each group: ☐

See answer on page 38

Why is it important? Tasks such as this provide an opportunity for learners to develop a deep and flexible understanding of mathematics. Grouping is foundational for understanding multiplication and division. When learners solve problems by grouping, they begin to see how numbers can be composed and decomposed, which is key for developing fluency and the use of efficient strategies. This problem also builds learners' skills of strategic thinking; rather than work through all the possible combinations, learners need to use their understanding of odd numbers to initially reduce the possibilities they try.

How difficult is this task? On average, 24% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? The components of this question (multiplication and division by 3 and 5, grouping, odd numbers) are all within the grasp of most Grade 4 learners. However, learners struggle with this question as the format is unfamiliar and presents a combination of cognitive, conceptual, and linguistic challenges. Firstly, learners need to have the understanding that grouping means to form equal sets, asking themselves, "How many times can I take a group of x out of 30?" Further, they need to understand that it is the group size, not the number of groups, which must be odd. Some learners will confuse these two, while others may try to find an answer where both the group size and the number of groups are odd, which is impossible. To avoid a trial-and-improvement approach, learners need to be fluent and flexible in their use of multiplication facts so that they recognize factors of 30 and identify possible solutions. Further, the language may be considered complex, as learners will need to interpret each phrase and hold the mathematical requirements of each fact in their head at one time.

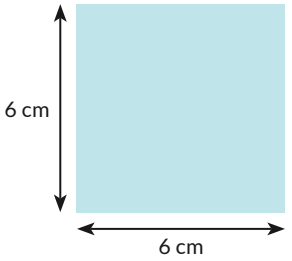
Didactic approaches:

- This task could be approached as a whole class on the interactive board or as a physical activity if space allows. Physically grouping 30 learners (or other objects) will enable learners to develop a deep understanding of what it means to group, as opposed to simply sharing.
- Whether working pictorially or physically, start with a question that includes just the first condition: all group sizes are the same. Establish that we want the same number of learners in each group and that we want to group all 30 learners. Ask learners for a suggestion and check whether this is correct or not, working it through physically. Emphasize that the number given is the number of learners in each group, so we remove that number of learners each time to create a group. Constantly emphasize the language of "creating groups of that number." If the number suggested by learners is a factor of 30 and equal grouping is possible, write the number sentence: There were 30 learners in all. I can put them into ☐ groups, with ☐ learners in each group. Ensure learners are clear about which number goes in each box and why. Use the language of the group size (e.g., 5) being a factor of

30—it divides into 30 equally. If the number suggested by learners is not a factor of 30 (e.g., 4), still work through the process of creating equal groups to show that a final equal group cannot be formed. Use the language of 4 not being a factor of 30; 4 does not divide into 30 equally.

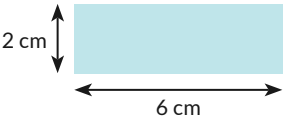
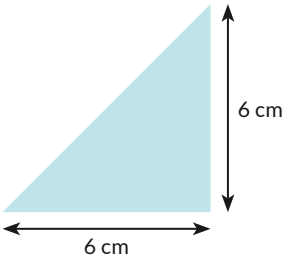
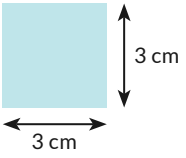
- Repeat this task to find other group sizes that can also be made. Support learners to develop systematic working skills and reasoning, explaining why certain numbers would not work. Write the number sentences in order to clearly illustrate all the factors of 30. Use the language: "These are factors of 30; 30 is a multiple of each of these numbers."
- Then, introduce the second condition: all group sizes must be odd. Ask learners which numbers we are interested in when looking at our number sentences. Be clear that we are focusing on the group size, not the number of groups. Circle those group sizes with an odd number (30 groups of 1; 10 groups of 3; 6 groups of 5; 2 groups of 15).
- Focus on the second statement and "odd numbers" being a mathematical property. Elicit other mathematical properties learners know (e.g., even numbers, square numbers, prime numbers, etc.). Learners should then work in pairs to create their own grouping problem with a condition statement such as in the original item. Have them swap questions between pairs and solve each other's puzzles.

8. Content Domain: Measurement and Geometry
Cognitive Domain: Applying
TIMSS Benchmark: Advanced
Description: Determines the number of three different shapes that cover the area of a square



The square above can be made by putting together smaller shapes.

Complete the table with the number of each shape that are needed to cover the whole square.

Shape	Number needed to cover the square above
	
	
	

See answer on page 39

Why is it important? Learners live in a spatial world. Understanding how to manipulate shapes has practical applications in everyday life, helping learners see the relevance of mathematics in the real world. Through such activities, learners develop spatial awareness, which is crucial for understanding the physical world. It allows learners to visualize and comprehend how objects fit together and interact in space. Shape manipulation is a fundamental aspect of learning geometry and measures, developing learners' understanding of properties of shapes, such as angles, sides, and symmetry, and how these properties relate to each other.

How difficult is this task? On average, 21% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? With a heavy focus on number in the primary school curriculum, many learners have quite limited experience in working with geometry and measures. Further, both digital technologies and a desire to record learners' work may lead to teachers moving towards pictorial representations before learners have had adequate experience with concrete materials. Imagining the transformation, rotation, and combination of shapes requires high levels of spatial awareness, combined with geometric knowledge and the ability to hold multiple ideas at once, hence drawing excessively on working memory.

Didactic approaches:

- Begin with a real-life application of the question. Engage learners in thinking about how they might place carpet tiles in a new reading area. If possible, mark out a 2 m x 2 m square with masking tape, but an image on the interactive whiteboard will also work well. Use carpet squares (50 cm square) and ask learners to investigate how they can use them to cover the larger area. Do not focus on measurements at this stage; instead, ensure learners understand how tiling works and that all edges must touch without overlapping.
- Provide learners with a range of gummed shapes (it may be sensible to limit the range). Ask them to explore how they can completely cover a given square using only one type of shape. Are there different ways they can do this?
- Provide learners with several squares of card. How many ways can they cut them up so the pieces are identical and can be reassembled to form a square? At this stage, they may need to measure rather than work by eye.
- Look at a square (with no measurements) on the board and ask learners what shapes could be used to cover it. Draw these in on separate diagrams (such as four smaller squares, two triangles, etc.). Now add in the dimensions of the square and ask learners what can be said about the dimensions of the smaller shapes tiled within it.
- Return to the original square and show the rectangle in Part 1 of the question. Where possible, use digital technology to help learners interact with the shapes and use color to distinguish between the shape to be filled and the shapes used to fill it.

Before learners engage interactively, ask them to discuss with a partner how many rectangles are needed and how they know. Test their responses. Repeat with the smaller square tiles.

- Repeat the above with triangular tiles. "Lock" the image so it cannot be rotated to highlight the nature of the problem. Unlock the triangle and show learners how the triangle can be rotated so that two triangles completely cover the square.
- Return again to the blank square that needs to be covered. This time, ask learners to consider which tile shapes will not fully cover the base square, drawing on their prior work with gummed shapes. This is an important step, as being able to reason and justify to others why a shape (such as a hexagon) tiles will not fill the square is developing the foundations of generalization.

9. Content Domain: Measurement and Geometry
Cognitive Domain: Applying
TIMSS Benchmark: Advanced
Description: Determines the number of square and triangular faces of three-dimensional shapes

Justin has many of these triangle and square panels that fit together to make three-dimensional shapes.

Justin makes each of the shapes shown in the table.

Fill in the table. The first one has been done for you.

Shape	Number of triangles	Number of squares
	4	1
	☐	☐
	☐	☐
	☐	☐

See answer on page 39

Why is it important? Mental rotation and three-dimensional visualization are key components of spatial reasoning, which is the ability to understand and manipulate objects in space. Strong spatial reasoning is associated with success across all areas of mathematics. Activities such as this, which require learners to visualize shapes from different viewpoints, support the development of their understanding of three-dimensional shapes.

How difficult is this task? On average, 27% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? At the age of 8–12 years old, learners' ability to visualize and manipulate shapes in their head is still developing. Their earlier opportunities to work on similar tasks may be limited. To be successful, learners will need to think flexibly, understanding that a three-dimensional shape can appear different from different angles and that a two-dimensional representation does not show the whole shape. They will also need to mentally rotate the shape and visualize it from different perspectives. The symbolic representations, such as a dashed line, may be unfamiliar to learners, and they may find it difficult to understand why the faces in a three-dimensional representation do not appear identical, nor are they similar in orientation or scale to the two-dimensional representations in the question.

Didactic approaches:

- To be successful in tasks such as this, learners need lots of practical experience with physical manipulatives as well as experience forming three-dimensional shapes from two-dimensional nets. Provide learners with manipulatives designed to support the construction of three-dimensional shapes, such as those where triangles and squares can be fitted together magnetically. Have learners work in groups. Begin with free play to ensure confidence in how the manipulative works and how the pieces fit together.
- Set groups a range of tasks to complete:
    - What can you make using only triangles? Can you find two examples? What can you say about the shapes you have made?
    - What can you make using only squares? Can you find two examples? What can you say about the shapes you have made?
    - What is the smallest number of shapes you need to make an enclosed three-dimensional shape? Why can you not make an enclosed shape with fewer two-dimensional shapes?
    - Can you make a cube? What does the cube look like when you flatten it out? Learners may need to be explicitly introduced to the language of nets.
- Provide groups with the first two images from the task (including the example). Ask them to try to replicate these in their groups and determine how many squares and how many triangles are required to produce each three-dimensional shape.

- Look at the next shape on the board. Without constructing it, ask learners to predict how many squares and triangles are required and to explain their reasoning. Whether you have consensus as a class or not, construct the shape together to establish the answer. Address any misconceptions that arose at the prediction stage.
- Learners should now complete the task in full. Discuss the final item to check understanding.
- As an extension, learners may go on to investigate nets of common three-dimensional shapes (cube, cuboid, tetrahedron, square-based pyramid, and cylinder) by either making the three-dimensional shapes and breaking them down into a two-dimensional net and tracing this or through being given nets drawn on card from which they should construct the three-dimensional shape.

10. Content Domain: Data
Cognitive Domain: Applying
TIMSS Benchmark: Advanced
Description: Determines the y-axis scale for a bar graph given the data in a table

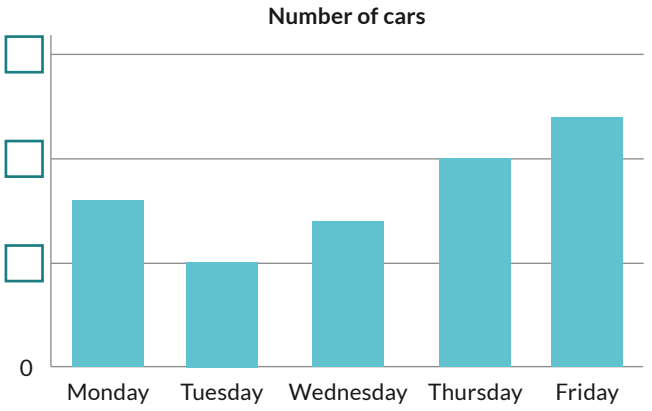
Skylar recorded the number of cars that travelled along her street each morning.

Day	Number of cars
Monday	8
Tuesday	5
Wednesday	7
Thursday	10
Friday	12

She started making a graph of her data.

What numbers should Skylar use to label the horizontal lines on her graph?

Put the numbers in the boxes on Skylar's graph.



See answer on page 40

Why is it important? The skills required to be successful in this item support learners' mathematical understanding and their ability to interpret and communicate data accurately. It is important that learners can interpret and construct graphs accurately, which includes an understanding of the scale to be used on either axis.

How difficult is this task? On average, 34% of Grade 4 TIMSS learners were able to complete this item successfully.

What makes it difficult? Most learners will be confident in counting in 1s, 2s, and 10s but may be less confident in counting in 5s. Even if confident in counting in 5s, they may find it difficult to estimate intermediary points, such as working out why the Monday bar is equal to 8. Learners may not have had experience with bar graphs using a y-axis scaled in intervals of 5 and may therefore apply more familiar scales—such as 1, 2, 3 or 10, 20, 30—without relating the scale appropriately to the data. The easiest way to tackle a question such as this is to start with the bars that sit on horizontal gridlines (Tuesday at 5 and Thursday at 10). Doing this requires experience and knowing that it is permissible to start with Tuesday rather than Monday. Once 5 and 10 are completed, learners will need an understanding of intervals to complete the 15. This may pose further issues as it is beyond the largest data point and learners may be insecure giving this answer.

Didactic approaches:

- This activity should not be attempted until learners have had significant experience with and are confident in producing bar charts from given data, selecting their own scale, and reading bar charts with different scales.
- It may be worthwhile for learners to have the data table, but not the semi-completed graph, and work individually or in pairs/small groups to construct their own bar chart from the data. This would serve as a useful assessment task to identify learners' current level of understanding and any misconceptions they may hold. Such misconceptions (which may include intervals only going up in 1s) should be addressed at this stage.
- Hopefully, in constructing their own graphs, learners will have used different scales (have some prepared with different scales in case this does not happen). Display two graphs (one using an interval of 1 and the other with intervals of 2) and ask which is correct. Elicit that they are both correct and simply show the data in a slightly different way. Discuss how the interval of 2 leads to a slightly more efficient graph, and ensure all learners are confident in how to read data with an odd number when that specific number does not appear on the scale (such as the data for Tuesday at 5). Repeat with a graph with an interval of 10.
- Show the task item. Explain that everything has been completed except the numbers on the scale. Remind learners of the previous task and that several intervals could be used, but they need to work out what interval has been used. Ask which data point should be started with. Many learners will want to start with the first point

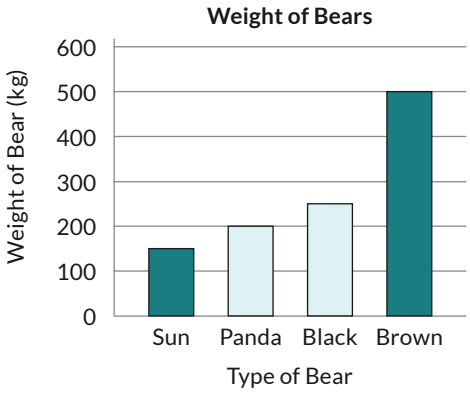
(Monday) as they are used to working left to right. Encourage them to consider whether another point would be easier. Draw their attention to Tuesday, where the data point is on the horizontal grid line. Looking at the data table, ask what we know this point to be (5), and complete this on the y-axis. Ask which other point could be completed in the same way (Thursday at 10). Knowing that the first point is 5 and the second point is 10, now ask what can be said about the interval being used (going up in 5s) and what the next point will be, even though we do not have any data to help us (15). Check with the remaining data point and discuss estimating points between grid lines.

- Depending on learners' confidence, this task could be repeated with a further set of data and a semi-constructed graph. Learners should experience a different interval (perhaps 10) so they do not develop the misconception that all missing interval questions have an interval of 5.

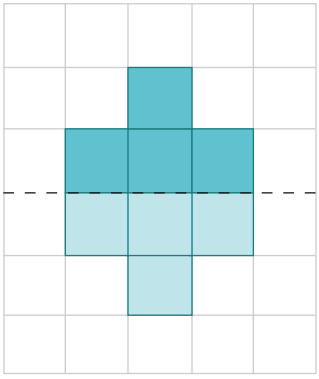
Answer Sheet

The answers shown illustrate the type of response that would receive full credit.

Item 1. Description: Represent data from a table in a bar graph
Answer:



Item 2. Description: Complete a symmetric figure on a square grid given half the shape and the line of symmetry
Answer:



Item 3. Description: Read data from a line graph
Answer: 16 m

Item 4. Description: Identify an expression that represents a situation
Answer: Ⓓ 12+(2x3)

Item 5. Description: Solve a word problem involving subtraction of a non-unit fraction from 1

Answer: $\frac{5}{8}$

Item 6. Description: Represent data from a table in a pictograph

Answer:

Animal	Weight (kg)
Cheetah	
Lion	
Leopard	

Key  =50 kg

Item 7. Description: Devise two ways of grouping objects that satisfy two conditions (2 of 2 points)

Answer:

Way 1

Number of groups: 6

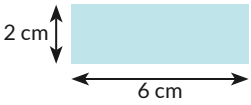
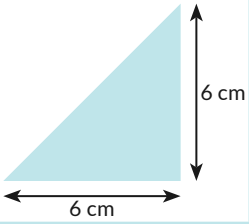
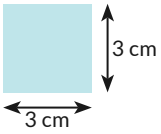
Number of children in each group: 5

Way 2

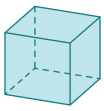
Number of groups: 10

Number of children in each group: 3

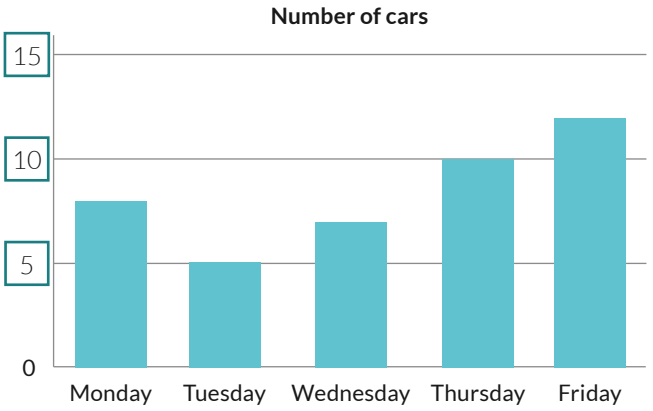
Item 8. Description: Determine the number of three different shapes that cover the area of a square (2 of 2 points)
Answer:

Shape	Number needed to cover the square above
	3
	2
	4

Item 9. Description: Determine the number of square and triangular faces of three-dimensional shapes (2 of 2 points)
Answer:

Shape	Number of triangles	Number of squares
	4	1
	4	0
	0	5
	2	3

Item 10. Description: Determine the y-axis scale for a bar graph given the data in a table
Answer:



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