Teaching and Learning Mathematics with Meaning: Using and Applying TIMSS

# Classroom Materials
# Grades 6-10

12 TIMSS items with didactic notes


**Content Domains:**
Algebra, Data and Probability, Geometry and Measurement, and Number

**Cognitive Domains:**
Applying, Knowing, and Reasoning

**TIMSS Benchmarks**
Low, Medium, High, and Advanced

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# Classroom materials for Grades 6-10

These materials consist of selected Trends in International Mathematics and Science Study (TIMSS) items, together with accompanying didactic notes to support their use in the classroom. These items were designed for Grade 8 learners participating in TIMSS assessments, but they are equally suitable for a range of other grades, depending on how the teacher uses them.

Each TIMSS item is presented on a separate page with descriptive information, with answers available in a separate sheet at the end. This information helps teachers identify items that align with their curriculum, learning objectives, or instructional focus. The descriptive information is as follows:

**Content Domain:**
TIMSS assesses Grade 8 mathematics across four content domains: *number*, *algebra*, *geometry and measurement*, and *data and probability*. Together, these domains represent the key areas of the lower secondary mathematics curriculum and measure students' understanding and application of fundamental mathematical concepts across increasingly complex contexts.

**Cognitive Domain:**
TIMSS assesses Grade 8 mathematics across three cognitive domains: *knowing*, *applying*, and *reasoning*. Together, these domains represent the cognitive processes that students develop as they engage with more advanced mathematics, measuring their ability to understand concepts, apply knowledge, and reason effectively to solve increasingly complex mathematical problems.

**TIMSS Benchmark:**
TIMSS reports student achievement at four international benchmarks: *Low*, *Intermediate*, *High*, and *Advanced*. These benchmarks describe increasing levels of mathematical knowledge and understanding.

**Description:**
The description of the task explains what students are asked to do and highlights the mathematical concepts and cognitive processes the task is designed to assess.

The accompanying **didactic notes** appear on the page following each item. They explain why the item is important and what makes it challenging and suggest practical teaching approaches and ideas for using the item in classroom practice.

Chapter 5 of *Teaching and Learning Mathematics with Meaning: Using and Applying TIMSS* (Golding and Marks; 2026) illustrates how a teacher might use one or more of these items in the classroom. However, a variety of other approaches are possible, and teachers will rightly choose to use them differently at different stages of learners' growing mathematical knowledge and confidence. What is important is that learners' mathematical *thinking* and *meaning-making* are focused on, rather than primarily the "correctness" of answers and the procedures used to obtain those answers.

The items, together with the percentage of students who answered each question correctly, may be found and downloaded for individual classroom use from the IEA TIMSS & PIRLS Website[1]. We suggest teachers work through the items themselves to understand the mathematical (and other, such as reading) demands involved.

[1] TIMSS 2019 International Results in Mathematics and Science (https://timss2019.org/reports/achievement/index.html). See Mathematics Grade 8 example items in the Performance at TIMSS International Benchmarks in Mathematics.

**1. Content Domain**: Number
**Cognitive Domain**: Applying
**TIMSS Benchmark**: Intermediate
**Description**: Find and compare unit prices ("zeds" are an imaginary currency used in TIMSS)

**Socks on Sale!**

Advertisements

| | |
|---|---|
| **SALE**<br>Store Q<br>6 pairs of socks<br>24.30 zeds | **SALE**<br>Store R<br>2 pairs of socks<br>8.40 zeds |
| **SALE**<br>Store S<br>4 pairs of socks<br>16.40 zeds | **SALE**<br>Store T<br>3 pairs of socks<br>12 zeds |

Chen has seen these advertisements for socks and wants to pay the lowest price per pair of socks. Complete the table below to show Chen the price per pair of socks in each store. Store Q has been done for you.

| Store | Price per Pair |
|---|---|
| Q | 4.05 zeds |
| R | zeds |
| S | zeds |
| T | zeds |

From which store should Chen buy her socks in order to pay the lowest price per pair?

Store:

See answer on page 32

**Why is it important?** Finding a unit price requires proportional thinking, which is central to many everyday and occupational tasks. Interpreting and applying that finding requires that learners make sense of their calculations, including an elementary understanding of decimal notation. Unit pricing is also used in many purchasing situations.

**How difficult is this task?** On average, 56% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** This is a very real context, but learners might not be used to engaging with ideas of "best value." Again, in many currencies, money is an authentic context for decimal work, but learners might not handle cash or budget for themselves. In some countries, experience with cash varies with learner background, so there may be related inclusion issues. Learners might have only a procedural grasp of decimal notation if they have been taught how to carry out decimal calculations but not how to estimate using decimals or make sense of those calculations. It is very common for learners to struggle to order decimals—for example, 4.8 and 4.09—correctly.

**Didactic approaches:**

- Have your learners had experience with such contexts using easier numbers, comparing just two situations? They might find it easier to start with their local currency.
- You could show learners just the pricing information: how could learners work out which socks are "best value"? Unit pricing is not the only way—you could ask for other ideas. If they were actually in this situation, would they need to do all of these calculations, or are there quicker ways to work out which is the cheapest per pair?
- You could use different pricing, where learners have to distinguish between commonly confused decimals such as 4.8 and 4.09. What (physical, drawn, or digital) decimal representations could you use that illustrate these decimals so that they are easier to compare? For example, learners might use 10 x 10 grids to represent 0.8 and 0.09 or make a habit of comparing decimals by writing all numbers with the same number of decimal places.
- Why might learners in fact not choose the shop with the lowest price per pair? Learners then begin to think about the limits of mathematics while making links with real life.

2. **Content Domain**: Number
**Cognitive Domain**: Knowing
**TIMSS Benchmark**: Intermediate
**Description**: Solve a word problem involving subtraction of negative numbers

On Thursday, the lowest temperature in City X was 6 °C and the lowest temperature in City Y was -3 °C. What was the difference between the lowest temperatures in the cities?

Answer ☐ °C

See answer on page 32

**Why is it important?** Incorporating negative numbers into their mental frameworks extends learners' grasp of the nature of number in ways that underpin later development of arithmetic and algebraic fluency.

**How difficult is this task?** On average, 59% of Grade 8 TIMSS learners were able to complete this item successfully.

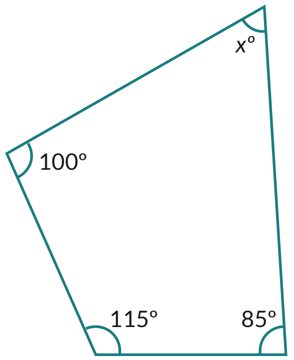
**What makes it difficult?** By their nature, negative numbers are not very tangible: they do not represent a physical "number of things" in the way that positive integers do, so they are not a central part of everyday conversation. Experience with genuine contexts for negative numbers, such as temperatures and budgeting, is therefore critical as a stepping stone towards representations on number lines and then further abstraction—though the example of low temperatures (on the given scale) might not be a direct experience for some learners.

Further, some aspects of the arithmetic behavior of negative numbers typically met later, such as their multiplication (and its inverse, division), are even more abstract. It is therefore important that learners are able to function confidently and fluently with core practical examples from which they can abstract.

**Didactic approaches:**

- Experience with informal language around temperature underpins more formal representations and procedures—it is important not to rush this.
- If concepts are not secure, use physical activity, for example, combining student jumps. Start with forward jumps: three jumps forward, then four jumps forward—how far have you jumped? (Seven jumps forward.) Now five jumps backward—how far have you moved from where you started? (Two jumps forward.) Now six jumps backward. How could you have reached the answer in one "go"? In parallel, represent these informally on the board. Choose a learner to decide the next jumping activity and to articulate what is happening.
- Formal representation on a number line is the next step towards abstraction.
- For teenagers in particular, money and budgeting provide another meaningful context, which again transfers easily to a number line.
- Once representation on a number line is secure, it makes sense to ask (or for learners to ask), "What is the gap between these numbers?" This is essentially what this question is asking.
- Only when representation is secure should you move to formal representation as a calculation.
- For formal calculations, mathematical cultures vary; most represent successive moves as above as additions—initially of two positive integers, then of a positive and a negative integer—but some elide the operation with the direction of the number. Whatever the advantages and disadvantages of local practice, it is important that learners are secure within it, building on a strong conceptual grasp.

**3. Content Domain**: Geometry
**Cognitive Domain**: Applying
**TIMSS Benchmark**: Intermediate
**Description**: Determine the value of an angle in an irregular quadrilateral given the values of the other angles




What is the value of $x$?

$x$=

See answer on page 32

**Why is it important?** Geometry is a natural context for deductive reasoning and is therefore critical to the development of problem-solving. Geometry is also deeply associated with learners' experiences of a two-dimensional and three-dimensional world and is of intrinsic interest in applications such as design, engineering, and art. We also know that cognitive and practical engagement with mathematical experiences in shape and space is central to learners' later and wider mathematical development.

**How difficult is this task?** On average, 56% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** Most learners are familiar with the angle sum of a triangle but are much less familiar with the angle sums of other polygons.

**Didactic approaches:**

- It is important that learners are easily able to "track back" to secure knowledge—here, the angle sum of a triangle. By joining two vertices of the quadrilateral, it is clear that the angle sum is double that of a triangle. A similar approach can be used for other polygons (for a challenge, learners can try applying this approach to a non-convex polygon).
- The associated mathematical vocabulary (vertex/vertices, edge, quadrilateral, interior/exterior angle, convex/concave) and the naming of angles using their contributing vertices are very empowering. This inducts learners into mathematical norms, helping them feel that they belong in a mathematical community. Such language can be introduced and reinforced gently through prompts such as "Can anyone remember…?", "Does anyone know…?", and "How would you say that in everyday language?"
- Some vocabulary is associated with the cultural roots of mathematics (for example, "quadrilateral" in English derives from the Latin for "four sides," whereas "isosceles" comes from the Greek for "equal legs"). Making such links with local language use further develops learners' identity as mathematicians and grounds them in a historical succession of mathematical activity.
- Angle sums can be explored using (free) dynamic geometry software, with conjectures made from the measurements reported in that software. Learners might be challenged to consider the extent to which such data represent "proof."
- Alternatively, if protractors or angle measurers are available, similar explorations can be conducted by asking learners to draw, for example, any pentagon and measure and sum the interior angles. The variety of shapes around the class offers a good range of examples, though the same issues arise. Learners often find such practical experiences highly engaging and rewarding and typically internalize the associated thinking at a deep level. While such approaches may appear time-consuming, they are both memorable and motivating for many learners.
- The curious might explore the angle sum of polygons on a globe (or balloon). For

example, what is the largest possible angle sum of a triangle drawn on the surface of a balloon? What does it mean to talk about an angle on a curved surface? Such horizon-extending questions can excite learners.

- It is also important to stress that an angle is a measure and, like all measures, must include units: here, x represents the number of degrees. Learners may be intrigued to know that other angle measures are used in fields such as engineering.

**4. Content Domain**: Number
**Cognitive Domain**: Applying
**TIMSS Benchmark**: High
**Description**: In a word problem dividing a quantity by a given ratio, determine the quantity of one of the parts

A piece of string was 45 cm long. Then, it was divided into two pieces in a ration of 4:5.

What is the length of the shorter piece of string in cm?

- Ⓐ 5
- Ⓑ 20
- Ⓒ 25
- Ⓓ 36

See answer on page 32

**Why is it important?** Proportional reasoning is a fairly advanced cognitive skill that builds on previous multiplicative reasoning. Its use is very common in a wide range of applications of mathematics, particularly within science. Division in a given ratio ("unfair sharing") is a particular example of proportional reasoning and, again, arises naturally in a range of contexts.

**How difficult is this task?** On average, 54% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** Learners need a robust understanding of ratio and proportion as multiplicative in nature rather than additive. Early examples are often given in terms of doubling, which allows for a retreat to additive or informal methods, so experience with a range of numerical ratios is important.

**Didactic approaches:**

- Learners need experience with a wide range of contexts for proportional reasoning and "unfair sharing" so that they appreciate the need for such thinking.
- Informal recording and the use of a bar diagram can build conceptual understanding.
- Learners can reflect on the reasoning they already use, examine proportion problems, and come to appreciate their multiplicative structure and create their own variants of proportion problems.
- It is helpful to expose and build from inappropriate additive reasoning (e.g., "What happens if…?") or informal methods.
- Support learners in comparing methods for solving a particular proportional reasoning problem. Discuss the relative strengths and weaknesses of the approaches used.
- Aim to move towards a single-step multiplicative calculation—for example, a 20 percent increase means finding "120 percent of …," which means multiplying by 1.20.
- This will be particularly important when finding reverse percentages—for example, "The final cost after a 20 percent increase was… What was the original cost?" This is solved by dividing by 1.20, not by subtracting 20 percent. Learners who are not fluent with using single-step multipliers will struggle with this concept (again, bars are useful representations).
- Remind learners that a proportional increase/decrease can be represented by a decimal multiplier. For example, to increase a quantity by 43 percent, multiply by 1.43; to decrease by 12 percent, multiply by 0.88 (ensure learners experience multipliers less than one).
- For the particular case of dividing in a given ratio, the notion of "unfair sharing" in different contexts can be helpful, as can a variety of informal recordings. This context offers vast opportunities to build, consolidate, and interrelate communication involving fractions, decimals, percentages, and ratios. A class-specific approach—such as dividing a large pile of imaginary chocolates unequally between a small number of volunteers—can be engaging.

**5. Content Domain**: Algebra
**Cognitive Domain**: Applying
**TIMSS Benchmark**: High
**Description**: Solve a word problem involving evaluating a formula with exponents

The stopping distance (*d*) meters depends on the speed (*v*) meters per second of the car when the brakes are applied. A formula for calculating this distance is:

$$d = \frac{2v + v^2}{20}$$

What is the stopping distance when *v*=20?

*d*= ______ m

See answer on page 32

**Why is it important?** This question focuses on algebraic notation, which is central to mathematical models. Learners will need to use formulae in science and other applications and might, for example, use spreadsheets to help them do so.

**How difficult is this task?** On average, 35% of Grade 8 TIMSS learners were able to complete this item successfully.

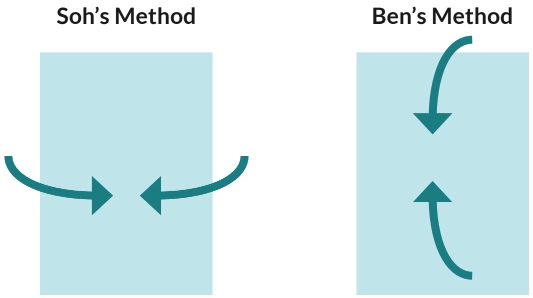
**What makes it difficult?** Learners need to understand the $v^2$ notation as representing v x v, conventions for the order of operations within algebraic notation, and the use of the fraction line as a bracket. They also need to understand how to operationalize substitution into a formula.

**Didactic approaches:**

- There is a wealth of experience needed before learners can confidently tackle questions such as these, building on an early "algebraic" grasp of number (appreciation of how numbers behave before any move to the use of letters). Some early approaches, such as "fruit salad" algebra, can undermine long-term goals, so make sure that letters in any formula always represent a number. Also, ensure learners gradually become confident in the associated language of formula, equation, variable, square, reciprocal, and so on, as this gives them confidence to communicate mathematically.
- Teach the conceptual ideas behind a mathematical formula or equation: allow learners to build their own formula or equation and ask their own related questions. Ask learners to explain in ordinary language what a formula is communicating and give them plenty of opportunity to input a range of values to check their understanding. As they mature, include decimals, fractions, and, where appropriate for the formula, negative numbers.
- Some common mathematical activities, such as finding perimeter or area, can easily be captured as formulae. Learners will encounter a range of formulae in science, for example, as they mature. Encourage them to recall and apply such experiences and to interpret their answers.
- Rigorous thinking around units related to the context will support learners' thinking and reinforce that within an equation or formula, any letter represents a pure number.

**6. Content Domain**: Geometry
**Cognitive Domain**: Reasoning
**TIMSS Benchmark**: High
**Description**: Compare properties of two open cylinders made by rolling the same rectangle in different directions

Soh and Ben have identical rectangular pieces of paper. They use different ways to roll their paper into cylinders so that the opposite sides of the paper touch as shown below.




Compare the properties of the two cylinders.
Use the drop down menus.

**Height**

Soh's cylinder [ ] Ben's cylinder

**Diameter**

Soh's cylinder [ ] Ben's cylinder

**Surface Area (with open ends)**

Soh's cylinder [ ] Ben's cylinder

See answer on page 32

**Why is it important?** Much mathematical work takes place within three-dimensional space, or within two-dimensional representations of that space, so understanding the relationships between the two is an important aspect of mathematical learning. Additionally, spatial visualization and reasoning contribute significantly to the foundations for a wide range of later mathematical functioning, well beyond the spatial domain.

**How difficult is this task?** On average, 41% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** Learners must not only visualize the process being undertaken and its outcome but also have secure concepts of diameter and surface area (which are preserved during the transformations described).

**Didactic approaches:**

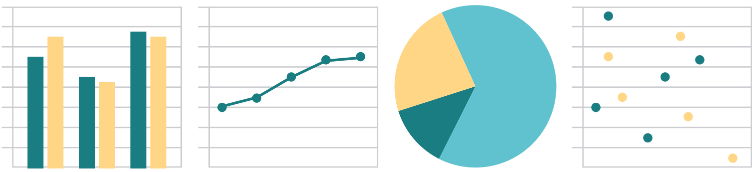
- Learners need a variety of experiences with handling two-dimensional and three-dimensional objects and with relating three-dimensional objects to two-dimensional representations; digital representations alone are insufficient.
- Experiences with paper folding, with physical patternmaking, and drawing a variety of representations of three-dimensional objects—including, if possible, on squared or dotted paper—are motivational for many learners and build spatial concepts. Digital manipulatives can complement, but not replace, these physical experiences.
- They also need physical experience with measuring (and estimating), building from informal measures to formal and careful use of appropriate units.
- “Reverse” tasks such as “draw a shape that has a perimeter of 12 cm” or “draw a shape that has an area of 12 $cm^2$,” followed by comparison and discussion, can help to deepen conceptual understanding of length, area, volume, angle, weight, time, and more.
- Alongside this, learners need to build a range of appropriate mathematical language: names of two-dimensional and three-dimensional shapes, vertex, edge, face, types of angles, parts of a circle, etc.
- As learners gain experience estimating and measuring, they can build a mental repertoire of “standard” measures. For example, they might recall that 1 kg is the weight of a standard bag of sugar, common bottles hold 500 ml, the classroom is about 7 m from front to back, or the distance from school to town is about 1 km—whatever local instances of measurement are meaningful.

**7. Content Domain**: Data and Probability
**Cognitive Domain**: Applying
**TIMSS Benchmark**: High
**Description**: Identify an appropriate graph for three different types of data

Lee wants to make three graphs showing information about his town.
The titles of his graphs are shown in the table below.

Which type of graph is best for each?

Drag one type of graph to each title.



| Job Types of Workers in Town | The Number of Girls and Boys Born Each Year | Town Population Over time |
|---|---|---|
| | | |

See answer on page 33

**Why is it important?** In our increasingly data-rich world, graphs and other representations of data are increasingly used to “make sense of” data—or, sometimes, to mislead with it. Learners need to engage with a variety of such representations, understand the strengths and limitations of each, and be able to critique how they are used.

**How difficult is this task?** On average, 47% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** Learners are asked not just to interpret data representations but to select from different possibilities the most appropriate type for a given situation. This requires understanding both the strengths and limitations of each representation.

**Didactic approaches:**

- Learners need experience with all aspects of a range of data representations: interpreting, critiquing, and using raw data to choose and construct their own representations.
- If facilities allow, learners can work with genuine digital datasets to construct digital representations. While this allows for rapid processing and representation of relatively large datasets, it does not substitute for experience of construction by hand, where learners have to make and justify their own choices and decisions about appropriate representation, scale, labeling of axes, and so on.
- It is motivational for learners to ask their own questions around data. For example: How are the height and arm-span of members of the class related? What data will they need to collect in order to answer that question? Having collected the data, how can they best represent it? What does the representation show?
- For the construction of pie charts, it is helpful if the number of data points is related to 360. For example, in a class of 26, 38, or 58, two learners might not have their data collected, but have specific other roles, or the teacher might be included if helpful to the arithmetic.
- The internet allows access to global information that learners might want to engage with, including for social justice education purposes. For example: How do average lifespans compare across the globe, and how might that be represented? What is the population density in parts of learners’ own countries? How is national gross domestic product spent, and is this fair?

**8. Content Domain**: Data and Probability
**Cognitive Domain**: Applying
**TIMSS Benchmark**: High
**Description**: Estimate the number of objects in a given probability sample

A bag contains 24 marbles, some white and some black.

A marble is chosen at random, its color is noted, and the marble is placed back into the bag. This is done 120 times, and a white marble appears 70 times.

How many white marbles are likely to be in the bag?

- Ⓐ 7
- Ⓑ 10
- Ⓒ 12
- Ⓓ 14

See answer on page 33

**Why is it important?** Many mathematical contexts that learners will need to engage with in their current or adult lives involve chance (probability), so it is important that they can engage confidently with related concepts.

**How difficult is this task?** On average, 43% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** Probability is by its nature abstract, and learners may feel they have little related experience. It is also expressed mathematically as a proportion (a fraction, decimal, percentage, or ratio). It is therefore an inherently demanding concept, needing confident engagement with ideas of both chance and fraction equivalence.

**Didactic approaches:**

- Learners need extensive discussion of ideas related to chance in a variety of contexts that are meaningful to them—for example, weather forecasts (particularly if they live in an area of uncertain weather), sporting events, or health outcomes. Are these situations “fair”? What would have to be altered to change those probabilities?
- The most concrete way of engaging with such ideas is through simulation, using dice or spinners, or digital equivalents. Learners can build experience of these—for example, by being chosen through a random mechanism to demonstrate the next solution on the board (named sticks or pieces of paper can be used)—and of the different ways in which we establish and quantify probabilities, including through equivalent fractions, decimals, percentages, and ratios.
- It is important that learners are gradually introduced to the related vocabulary, without which they cannot succinctly communicate the related ideas (random, equally likely, fair, etc.).
- Ensure also that learners have the opportunity to come to understand the difference between such situations “with” and “without” replacement (this relates easily to the in-class random selection of a name).

**9. Content Domain**: Number
**Cognitive Domain**: Reasoning
**TIMSS Benchmark**: Advanced
**Description**: Solve a multistep problem involving addition and subtraction of fractions

In the square below:

- The numbers in each row add to 1,
- The numbers in each column add to 1, and
- The numbers in both diagonals add to 1.

| | | |
|---|---|---|
| $\frac{8}{15}$ | | $\frac{2}{5}$ |
| $\frac{1}{5}$ | $X$ | |
| | | |

What is the value of *X*?

*X*= $\boxed{\;—\;}$

See answer on page 33

**Why is it important?** This task involves a string of deductive reasoning but also an expanded and fluent conception of real number, beyond the positive integers.

**How difficult is this task?** On average, 18% of Grade 8 TIMSS learners were able to complete this item successfully.

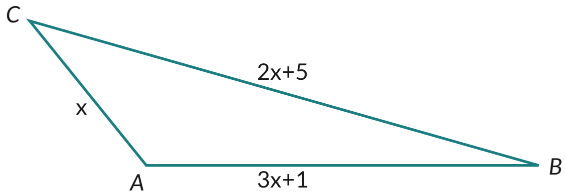
**What makes it difficult?** The task is multistep and unstructured but also involves multiple additions of fractions with different denominators.

**Didactic approaches:**

- In theory, this task could be accomplished by any learner who can follow the instructions and add fractions with different denominators. The task could be given to individuals or to small groups within the class.
- The difficulty depends in part on the extent of previous experiences learners have had with these "magic squares." They might well have encountered magic squares populated by positive integers and be intrigued to see ancient documents and illustrations that include them (the first recorded magic square is the Lo Shu Square, found in ancient China about 5,000 years ago).
- If they find the fraction square demanding—or even if they do not—they could tackle squares with other classes of numbers as entries, including positive or negative integers, decimals, mixed numbers, or algebraic expressions (including algebraic fractions). Who can create the most interesting magic square?
- Dimensions need not be restricted to 3 x 3, but in a 3 x 3 magic square, is there something interesting happening with the center entry? Will that always happen? How do learners know? There is an opportunity for development of ideas of proof here.

**10. Content Domain**: Algebra
**Cognitive Domain**: Applying
**TIMSS Benchmark**: Advanced
**Description**: Construct a linear equation for the perimeter of a triangle and solve for the length of one side

The perimeter of triangle ABC is 21 cm.




What is the value of *x*?

$x=$ ☐ cm

See answer on page 33

**Why is it important?** Many real-world mathematical tasks are initially unstructured, requiring the formation of an equation as a mathematical model representing known constraint(s), and then the solution of that equation (or equations) and interpretation of the result(s).

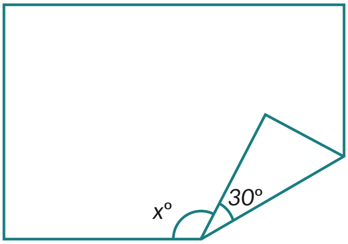
**How difficult is this task?** On average, 26% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** This task requires the learner to insert structure and then solve a linear equation. The value for x is not an integer, which might threaten the confidence of less experienced learners.

**Didactic approaches:**

- Too often, textbook exercises and examination questions are tightly structured, supporting the easy allocation of marks. Learners also need to experience—and have value placed upon—a variety of mathematical situations where they must choose the structure and mathematize the situation from the information given.
- Initially, they might do this through whole-class discussion or paired or group work to support both confidence and the articulation of thinking.
- The solution of fairly extended linear equations such as the one arising here can be supported through the annotation of each step metacognitively: “The perimeter is the sum of the lengths of all the sides”; “Simplify each side”; “Collect all the x terms on one side and the numbers on the other”; “Divide both sides by six.” Such metacognition supports conceptual reminders of the justification for each step and the generalization of the approach used.
- Ensure that, over time, learners experience solving a range of equations where the unknown is not an integer. Here, of course, since x represents the number of units in a length, x must be positive; in other models it might be negative, and learners should experience (and interpret) such outcomes.

**11. Content Domain**: Geometry
**Cognitive Domain**: Reasoning
**TIMSS Benchmark**: Advanced
**Description**: Use properties of supplementary angles to solve for an angle




A rectangular piece of paper is folded at one corner, as shown above.

What is the value of *x*?

Answer:

See answer on page 33

**Why is it important?** This task requires following through several steps of deductive reasoning, a core skill in many applications of mathematics.

**How difficult is this task?** On average, 26% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** The task, like many real-world mathematical tasks, is an unstructured one on which learners have to impose structure. They must also interpret the diagram as representing a previous fold—so that the 30-degree angle is superimposed on an identical angle—and follow through on the implications. Each step is straightforward if structured for learners, but taken together and without support, the task becomes much more demanding.

**Didactic approaches:**

- Learners need to experience multiple instances of moving between three-dimensional situations (here, the fold takes us into the third dimension) and two-dimensional representations of those situations.
- Many learners will be able to find the answer but struggle to communicate the reasoning that led them to that answer. This can be supported through whole-class discussion, careful teacher questioning, and teacher modeling of robust ways to communicate the steps involved. Tasks could then move to group or paired work, so that learners have peer support and must communicate their thinking orally before committing it to writing.
- Such tasks can be extended to a variety of geometric situations, including wider polygonal, circle, and three-dimensional geometries, and learners can create their own questions.

**12. Content Domain**: Data and Probability
**Cognitive Domain**: Applying
**TIMSS Benchmark**: Advanced
**Description**: Determine the change in a mean given changes in individual scores

A relay team for a 400 m race has 4 runners. They took 12 seconds, 13 seconds, 11 seconds, and 13 seconds, respectively, to complete their legs of the race.

In the next race, 2 of the runners each improved their times by 2 seconds, and the other 2 had the same time as before. By how many seconds did the team's mean running time improve?

Ⓐ 0 sec.

Ⓑ 1 sec.

Ⓒ 2 sec.

Ⓓ 4 sec.

See answer on page 33

**Why is it important?** In an increasingly data-rich world, indicators of “typical” data, and their spread, are often more useful than the raw data. Learners need to be confident in selecting, applying, and interpreting a variety of such measures—including in situations where there is more information given than needed or, conversely, where the information seems insufficient.

**How difficult is this task?** On average, 36% of Grade 8 TIMSS learners were able to complete this item successfully.

**What makes it difficult?** Many textbook and examination questions focus on closed procedures that test learners’ knowledge of definitions (for example, mean, median, or mode) but not their understanding of the core concepts underlying those definitions. In this example, the original mean time is not an integral number of seconds (nor is it actually needed), there is additional irrelevant information, and the task also lacks information some learners might feel they need (for example, which two runners improved their time).

**Didactic approaches:**

- As with much core mathematical vocabulary, there is much to be gained by asking learners to explain in everyday language what “the mean” represents—not how to calculate it, but what it signifies. For example, if all the runners ran the mean time, the total time taken would be the same. The total time taken is therefore intrinsic to a conceptual grasp of the mean.
- There is also great benefit in offering learners experience of both “direct” tasks (finding the mean, median, or mode of a set of data, especially large datasets) and “reverse” tasks. For example:
    - Find four numbers whose mean is 15.
    - Find four numbers whose mean is 15 and whose mode is 13.
    - Find four numbers whose mean is 15, whose mode is 13, and whose median is 14.5.
- It is important for learners to experience situations in which there is not a single correct answer and to become fluent in both direct and inverse processes.
- In this context, there are a variety of approaches that are valid—some of which are quicker, easier, or more “elegant” than others. Discuss, question, and value all valid approaches in class. Ask learners whether that approach will always work and why (not) so that learners build their confidence and mathematical identity.

## Answer Sheet

*The answers shown illustrate the type of response that would receive full credit.*

**Item 1. Description**: Find and compare unit prices ("zeds" are an imaginary currency used in TIMSS)
Answer:

Store: T

**Item 2. Description**: Solve a word problem involving subtraction of negative numbers

Answer: 9 °C

**Item 3. Description**: Determine the value of an angle in an irregular quadrilateral given the values of the other angles
Answer:

$x =$ 60

**Item 4. Description**: In a word problem dividing a quantity by a given ratio, determine the quantity of one of the parts
Answer: Ⓑ 20

**Item 5. Description**: Solve a word problem involving evaluating a formula with exponents
Answer:

$d =$ 22 m

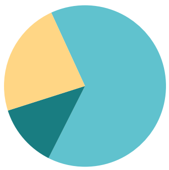
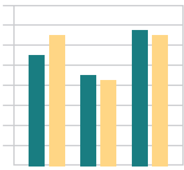
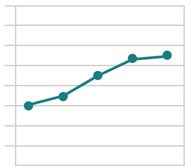
**Item 6. Description**: Compare properties of two open cylinders made by rolling the same rectangle in different directions
Answer:

**Height**
Soh's cylinder > Ben's cylinder

**Diameter**
Soh's cylinder < Ben's cylinder

**Surface Area (with open ends)**
Soh's cylinder = Ben's cylinder

**Item 7. Description**: Identify an appropriate graph for three different types of data
Answer:

| Job Types of Workers in Town | The Number of Girls and Boys Born Each Year | Town Population Over time |
|---|---|---|
|  |  |  |

**Item 8. Description**: Estimate the number of objects in a given probability sample
Answer: Ⓓ 14

**Item 9. Description:** Solve a multistep problem involving addition and subtraction of fractions
Answer:

$x = \boxed{\frac{5}{15}}$

**Item 10. Description**: Construct a linear equation for the perimeter of a triangle and solves for the length of one side
Answer:

$x = \boxed{2.5}$ cm

**Item 11. Description**: Use properties of supplementary angles to solve for an angle
Answer: $\boxed{120}$

**Item 12. Description**: Determine the change in a mean given changes in individual scores
Answer: Ⓑ 1 sec

**Source**: Golding, J., Marks, R. (2026). *Teaching and Learning Mathematics with Meaning. IEA Research for Educators*, vol 4. Springer, Cham. https://doi.org/10.1007/978-3-032-35352-8