DIFFERENCES IN TEACHING AND LEARNING MATHEMATICS IN CLASSES OVER THE WORLD: THE APPLICATION OF ADAPTED LEADERS CLUSTERING METHOD

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BACKGROUND

The Purpose of the Study

Mathematics, in general, is culturally independent. But is teaching mathematics also culturally independent? And if so, to what extent? Trends in International Mathematics and Science Study, TIMSS, collected enough information from students and teachers to find some answers to those questions.

The initial idea was to compare mathematics teachers from all participating countries regarding their teaching practices and then study the effects of differences in teaching approaches on the mathematics achievement of students. Since in TIMSS, along with teachers, students were also asked about their activities and those of their teacher's during mathematics lessons, the teacher's work in class is also described by his/her students' reports. Naturally, we wanted to join the teacher's report about his/her work in the classroom with the corresponding student reports about the teacher's work, but keep teachers as the units of the analysis. Also we had to take into account that students' as well as teachers' answers to most of the questions could not be treated as ordinal variables. For the analysis of our data, the most appropriate statistical methods, which meet both limitations, are Clustering Methods for Symbolic Objects.

We will present an application of the Adapted Leaders Clustering Method (ALCM), which was recently developed and is available as a software package ClaMix (Korenjak-Černe). We used the ALCM on data concerning all mathematics teachers and their students collected in TIMSS, Trend in Mathematics and Science Study, 1999, from 37 participating countries.

The Data

As it is mentioned before, we tried to link teacher variables with student variables. All teachers and all students together were regarded as a kind of ego-centered social network. In ego-centered networks in general every person, i.e. ego, is described by values of variables describing him and by values of other variables talking about his/her friends or colleagues, his/her alters. Usually, egos report about alters. In our case, teachers are egos, linked to their students who are regarded as alters, and alters, students, report about egos, teachers. In usual networks there are also relations between different egos but our teachers from all countries involved were not treated as being related to each other.

In order to use Clustering Methods to group similar teachers into a small number of groups, which could be further compared, we had to define the relation of similarity or dissimilarity between teachers taking into account not-ordinal nature of available variables.

In TIMSS, many questions for students and teachers had response categories, for which a calculation of means of a group of people's answers is not appropriate. For example, if the available response categories were 'always', 'sometimes' and 'never', coded respectively by 3, 2 and 1, the mean of 2.3 is quite difficult to interpret. It would be much easier to study a distribution of all answers in that group of people, e.g., frequencies of responses. In symbolic data analysis approach, we avoid means but keep the whole information on distributions of responses. For that reason we defined teachers as symbolic objects, described by values of a set of teacher variables and distributions of values of a set of student variables. Dissimilarities between clusters in such cases could be calculated using the Adapted Ward's Method for symbolic objects.

Symbolic Descriptions of the Data

When teachers are regarded as symbolic objects, variables describing them are combined from the variables of the teachers' dataset and the variables of the students' dataset in the following way:

1. Each response category of the teacher variable is replaced with its sequence number.

Example: *Q. What is the highest level of education you have completed?*

1. secondary school
2. BA or equivalent -> *TEDUC = 2*
3. MA / PhD

If a teacher selected the answer BA or equivalent, the variable TEDUC associated with question Q gets the value 2.

2. For each teacher, the variables of his/her students are presented with relative distributions.

Let's denote the student's variable by S and assume that it has k_S possible response categories. Then in the symbolic description of teacher A, the variable S is described with the vector $f(A;S)=[f(1, A;S), f(2, A;S), ..., f(k_S, A;S)]$ of the relative frequencies of student responses by categories; that is $f(j, A;S)$ is the number of responses of category j by the students of teacher A, divided by the number of all students of teacher A. The sum of all $f(j,A;S)$ over all possible answers j (j=1, 2, ..., k_S) on question S is 1.

The description of a teacher then consists of the teacher id, the vector of the teacher answers (replaced by the appropriate sequence numbers of them) and vectors of distributions of student variables.

Example: The symbolic description of the selected teacher whose id is 4567

Teacher id	Teacher variables - seq. num. of categories of responses					Distributions of values of students' variables		
↓	↓					↓		
4567	*2*	*4*	*6*	*8*	...	*[0.20,0.25,0.45,0.10]*	*[0,0,1]*	...
↑	↑	↑	↑	↑		↑	↑	
id	T_1	T_2	T_3	T_4		S_1	S_2	

In the program CLAMIX each teacher's variable T is also described with distribution $f(A;T)=[f(1,A;T), f(2,A;T), ..., f(k_S,A;T)]$, where $f(t,A;T) = 1$ if A checked t-th category at question T, and $f(j,A;T) = 0$ for all other j ($j \neq t$). Finally, symbolic object for teacher A is formed from the teacher and student variables and it is the vector $So(A)=[f(A; T_1), ..., f(A; T_t), f(A; S_1), ..., f(A; S_s)]$.

The Dissimilarity

The dissimilarity between teachers A and B is the sum of the dissimilarities on each variable

$$d(A,B) = \sum_{j=1}^{m} d(A,B;V_j),$$

where the dissimilarity on each variable is calculated as a half of the sum of square differences between components of corresponding symbolic objects:

$$d(A,B;V_j) = \frac{1}{2}\sum_{i=1}^{k_j}(f(i,A;V_j) - f(i,B;V_j))^2,$$ where k_j denotes the number of all respondent categories of the variable V_j.

The Optimization Problem

In our case, the units of the analysis are teachers, described as symbolic objects. We want to assign them to disjoint groups in such a way that the members of each group will be as comparable as possible regarding the previously defined dissimilarity. Among all feasible clusterings, which are all possible partitions of the set of all teachers, we want to find the one where cluster members will be the least dissimilar with respect to the criterion function. We are looking for the partition on which the value of the criterion function would be the smallest. Mathematically, clustering is defined as an optimization problem: In a set of all feasible clusterings Φ, and for the criterion function P, find a clustering $\boldsymbol{C}^*$ where $P(\boldsymbol{C}^*) = \min\{P(\boldsymbol{C}); \boldsymbol{C} \in \Phi\}$. Value of the criterion function on clustering is the sum of the clusters' errors: $P(\boldsymbol{C}) = \sum_{C \in \boldsymbol{C}} p(C)$. Cluster error *p(C)* is defined as the sum of the dissimilarities between each cluster's unit X and the leader L_C of the cluster *C,* $p(C) = \sum_{X \in C} d(X, L_C)$. The leader of a cluster is such a symbolic object that the sum of dissimilarities between this object and each cluster member proves minimal.

The Adapted Leaders Method

It is usually impossible to calculate a criterion function for all feasible clusterings and then choose the best clustering because of a huge number of feasible clusterings. Instead, to find the best solution we use a heuristic, Leaders Method Adapted to Symbolic Objects, ALCM. The method can, from an initial clustering, find a local optimal clustering with the requested number of clusters. The initial clustering could be determined in advance or randomly generated. The Leaders Method can be described with the following scheme:

Determine an initial clustering
Repeat
  Determine leaders of the clusters
  Assign each unit to the nearest leader
Until the leaders stop changing

The description of leaders in ALCM is also a symbolic object that minimizes the cluster's error:
L_C =[$s(L_C ; T_1), ..., s(L_C ; T_t), s(L_C ; S_1), ..., s(L_C ; S_s)$].
Here each variable $V = T_j$ or $V = S_j$ is described with the distribution
$s(L_C ;V)$=[$s(1, L_C ;V), s(2, L_C ;V), ..., s(k_V, L_C ;V)$], where

$$s(i, L_C; V) = \frac{1}{card(C)} \sum_{X \in C} f(i, X; V).$$

As a result of implemented program ClaMix, based on this method, we get lists of members of each cluster, descriptions of cluster leaders, value of criterion function of the whole clustering and values of cluster errors. Besides, for each cluster, the list of characteristic variables and their values is presented, called the description of a cluster. It contains variables for which more than a requested percentage of cluster members chose the same respondent category. For example, if we choose the characteristic percentage to be more than 60%, with 80 % of male cluster members, a variable associated with gender would be a characteristic variable for this cluster. Descriptions of clusters are the basis for interpretation of cluster characteristics.

At the next step, because of extended list of clustering results, it is possible to build a hierarchy of clusters to find out which clusters as a whole are more similar between themselves then others.

Building a Hierarchy with the Adapted Ward's Method

To reveal the internal structure of the clustering, the Adapted Ward's Method is used, which is a variant of the Agglomerative Clustering Method. The Agglomerative Clustering Method can be described with the following scheme:

Initial clusters are at level 0
Repeat
  Determine the closest pair of the clusters in the clustering
  Join them
  Determine the level of the new cluster
Until only one cluster remains

The level of the cluster $C = C_1 \cup C_2$ is defined by the dissimilarity between the joined clusters C_1 and C_2, from which the new cluster is formed. The dissimilarity between clusters is determined with the Ward's Relation

$$D(C_1, C_2) = \frac{card(C_1) \cdot card(C_2)}{card(C_1) + card(C_2)} d(L_1, L_2),$$

where $card(C_i)$ is the number of teachers in the cluster C_i and L_i is the leader (the representative element) of the cluster C_i.

The final output of the procedure could be directly used to draw a hierarchical tree, which graphically shows us the structure of a clustering. The distance between the initial level 0 and the level of the joined cluster $C = C_1 \cup C_2$ is equal to the dissimilarity $D(C_1, C_2)$ of clusters C_1 and C_2.

RESULTS

As stated at the beginning, we used TIMSS 1999 student and teacher data. In two of the participating countries teachers had too many missing values so we decided not to include them into our analysis. The following table shows the number of included teachers from each country.

Table 1: Number of Teachers Included in the Study

Number of teachers					
Australia	173	Iran, Islamic Republic	170	Philippines	150
Belgium (Flemish)	281	Israel	326	Romania	147
Bulgaria	164	Italy	183	Russian Federation	189
Canada	396	Japan	146	Singapore	145
Malaysia	153	Jordan	147	Slovak Republic	148
Chinese Taipei	151	Republic of Korea	193	Slovenia	152
Cyprus	122	Latvia	152	South Africa	199
Czech Republic	148	Republic of Macedonia	149	Thailand	150
England	539	Malaysia	153	Tunisia	149
Finland	180	Republic of Moldova	150	Turkey	204
Hong Kong	151	Morocco	349	United States	471
Hungary	195	Netherlands	126		
Indonesia	150	New Zealand	159	Total	7345

We included into the analysis all available internationally comparable variables which measure teacher's or student's opinions about mathematics as a school subject and variables measuring activities in mathematics lessons. The whole list of included variables is given in Appendix.

Despite the fact that among variables used in the analysis there was no variable that reported the country or cultural background of specific teachers, the analysis of cluster members by countries gave us interesting results. For all countries, the majority of teachers were assigned to one of the clusters, which means that within countries teachers are very similar.

We observe that countries having the majority of teachers in the same cluster share similarities in language, cultural or historical background, independently of the factors we studied in our analysis. All participating Eastern and Central European countries are associated with cluster 6, countries with English as the official language are associated with cluster 7. Results are shown in the following table.

Table 2: Clusters of similar teachers by countries

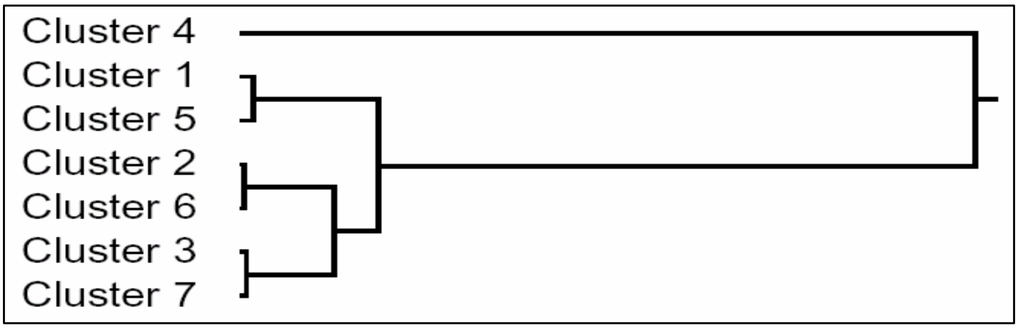
Countries	Percentages of teachers assigned to each cluster						
	Clus.1	Clus.2	Clus.3	Clus.4	Clus.5	Clus.6	Clus.7
Morocco	**86.5**	2.6	.9	5.4	3.4	0.9	0.3
South Africa	3.0	**64.8**	7.0	3.0	13.6	1.5	7.0
Indonesia		**58.0**	4.0		37.3	0.7	
Jordan	1.4	**55.1**	1.4	0.7	36.1	4.8	0.7
Chile	1.6	**47.9**	6.9		35.1	6.4	2.1
Republic of Moldova		**46.7**	0.7	2.7	22.7	26.7	0.7
Italy		**43.7**	3.8	0.5	15.3	36.1	0.5
Belgium (Fl.)		1.1	**79.4**	6.8	2.8	6.0	3.9
Hong Kong	3.3	4.6	**71.5**		1.3	6.6	12.6
Netherlands			**70.6**	10.3		1.6	17.5
Japan	3.4	4.8	34.2	1.4	**47.9**	8.2	
Republic of Korea,	1.6	6.2	7.3		**67.9**	17.1	
Chinese Taipei	0.7	27.2	10.6		**47.0**	14.6	
Malaysia	1.3	28.8	1.3		**67.3**	1.3	
Thailand		32.0	3.3		**64.0**	0.7	
Philippines		41.3	0.7	0.7	**57.3**		
Macedonia, Rep. of	0.7	29.5	1.3	0.7	**49.7**	10.7	7.4
Romania	0.7	12.2	2.7	0.7	**62.6**	21.1	
Turkey		24.0	12.7	1.0	**58.3**	3.4	0.5
Cyprus	1.6	27.0	.8	23.0	**29.5**	18.0	
Tunisia		31.5	20.8	2.0	**38.9**	6.7	
Iran, Islamic Rep.	0.6	31.2	3.5	0.6	**61.2**	2.9	
Russian Federation		11.1	0.5		1.1	**87.3**	
Slovak Republic	2.7	0.7	4.7	5.4	3.4	**83.1**	
Czech Republic	3.4		12.8	4.7	2.7	**75.7**	0.7
Hungary	0.5	2.1	7.7	2.1	22.6	**64.1**	1.0
Bulgaria	0.6	19.5	6.7	2.4	8.5	**62.2**	
Slovenia	2.0	2.6	3.9	2.0	28.3	**60.5**	0.7
Latvia	5.9	7.2	3.3	5.3	32.2	**46.1**	
Finland	8.3	1.1	33.3	0.6	3.9	**41.7**	11.1
New Zealand	0.6	1.9	25.2	0.6	3.1		**68.6**
Singapore		9.7	23.4	0.7			**66.2**

Countries	Percentages of teachers assigned to each cluster						
	Clus.1	Clus.2	Clus.3	Clus.4	Clus.5	Clus.6	Clus.7
USA	4.2	5.5	8.3	14.2	4.0		**63.7**
Canada		2.3	11.9	20.2	1.8	0.5	**63.4**
Australia	0.6	1.7	28.3	8.7	4.6	1.7	**54.3**
England	1.9	0.4	7.2	38.8	0.2	0.2	**51.4**
Israel	1.8	20.6	20.9	10.4	4.3	13.8	**28.2**
Total	5.6	16.5	14.5	7.4	21.1	16.8	18.1

Hierarchy of Clusters

The result of building the hierarchical tree is shown in the following diagram, together with a table of countries in their association to clusters.

Figure 1: Hierarchy of clusters



Cluster 4	Cluster 1	Cluster 5	Cluster 2	Cluster 6	Cluster 3	Cluster 7
Missing data	MOR	JAP, KOR, ROM, TUR, CYP, CTP, MYS, MKD, IRN, TUN, THA, PHI	SAR, IDN, JOR, CHI, MOL, ITA	RUS, SLO, CZE, HUN, BUL, SVN, LAT, FIN	BEL, HKG, NLD	NZL, SGP, USA, CAN, AUS, ENG, ISR

Cluster 4 with teachers having a majority of missing values is least similar to all other clusters. Cluster 2 and 6 are the most similar, then cluster 3 and 7, followed by clusters 1 and 5. Joined clusters 2 and 6 are the most similar to joined clusters 3 and 7. Then, joined clusters 1 and 5 are joined with all other clusters except 4, which stays at far distance until the last step.

The results of association of countries to clusters should be carefully interpreted. Teachers are not representative samples of all teachers from participating countries.

Clusters and Achievement

Students were assigned by achievement to percentile levels (25., 50., 75., 90) within a country and overall. The variable of national percentile levels for each student was included into the formation of clustering. It was assumed that teaching activities between teachers of two classes of high achievers are more similar between themselves than teaching activities in one low achieving class of students and one high achieving class of students. So, teachers inside a country having students with similar distributions of percentile levels are assumed to be more similar between themselves as they would be, if their students had very different distributions of achieved percentile levels.

The variable of international achievement was not included in the analysis as we wanted to avoid the assumption of high similarities between highest achieving countries. International percentile levels by clusters were analyzed after the clusterings were found. For all teachers the average percentage of students reaching each percentile level was calculated. Results are presented in Table 3.

Percentages of students associated with each cluster, who reached different percentile levels, quite differ from cluster to cluster. Cluster 1 has a large proportion of students at the first level and a very small proportion of students reaching 90th percentile.

Cluster 2 has a similar pattern, more (38%) than average (22%) percentage of students did not reach 25th percentile, a little above average reached second level and under average percentages reached other 3 levels.

Table 3: Comparison of Student Achievement Among Clusters of Their Teachers

	Mean percentages of students by their TIMSS test score, who internationally…				
Clusters	...did not reach 25th percentile	...reached 25th percentile	...reached 50th percentile	...reached 75th percentile	...reached 90th percentile
1	58.76	19.19	4.90	1.23	0.40
2	38.33	26.60	19.70	9.68	5.69
3	8.84	16.54	28.25	25.48	20.89
4	16.41	24.69	31.21	17.08	7.70
5	26.70	26.51	23.08	13.10	10.61
6	8.27	21.70	32.92	22.02	15.09
7	9.10	22.83	31.96	21.85	14.26
Total	21.79	23.47	25.67	16.59	11.79

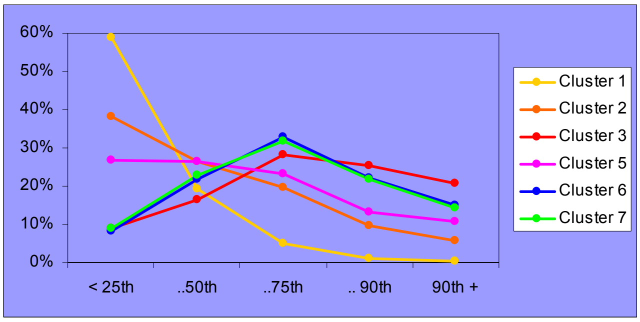
Cluster 3 has the highest achievement. The percentage of students who didn't reach 25th percentile is the smallest among all clusters, and percentages of students in the two highest percentile levels are the largest of all clusters. Twice as many students above the average reached 90th percentile. Result show that teachers in cluster 3 are the most effective teachers.

Cluster 4 joined teachers with missing data, and here we see that these teachers are well distributed among all levels. Cluster 5 students are just slightly worse than average; cluster 5 has a little more than the average number of students at first two levels and little less than the average number of students at the three higher levels.

Cluster 6 and 7 are similar in pattern. They both have half less students at the lowest level than average, and a third more students reaching 50th percentile than average. Despite small differences, cluster 6 is by achievement slightly better than 7.

In general, clusters differ quite a lot by achievement of corresponding students. In general, which class activities and teaching approaches could be associated with these differences is extractable from descriptions of cluster characteristics.

Percentages of students in clusters by their mathematics achievement



Description of Cluster Characteristics

Teachers in a cluster were similar among themselves, if they answered the questions similarly. We expect that a large number of cluster members chose the same answers to the questionnaire and that a large percentage of those teachers' students also chose the same answers to their questions. Extracted descriptions of clusters prove this to be true. For each cluster, there is a list of characteristic variables which have the same value for a large percentage of cluster members.

In general, there are some variables which are high on the list for all clusters, for example variables about the importance of students' knowledge and skills in order to be good in mathematics, but the range of importance is quite different among clusters. The following tables contain information about characteristic variables and their values for all clusters.

Cluster 1

The teacher's use of computers in cluster 1 is very rare, as 89.3% of teachers of that cluster have never used a computer for the purpose of teaching mathematics.

83.7% of teachers said it is very important for students to be good at math in order to be able to think in a sequential and procedural manner. It is followed by importance of students' understanding of concepts and principles, finding reasons to support solutions and understanding how mathematics is used in real world.

This is the only cluster where education is among the characteristic factors. Two thirds of teachers in cluster 1 only completed secondary school.

Characteristics of Cluster 1		
Percentage of same answers	Variables	Variable value
89.27%	Btbmask4: How often do you ask students to use computers?	never
83.66%	Btbmimp2: Teacher opinion: To be good in math - how important is it for students to think in a sequential and procedural manner?	very important
77.80%	Btbmimp3: Teacher opinion: To be good in math - how important is it for students to understand concepts and principles?	very important
71.46%	Btbmimp6: Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	very important
67.80%	Btbmimp5: Teacher opinion: To be good in math how - important is it for students to know how math is used in the real world?	very important
66.59%	Btbgeduc: Teacher's highest level of education	secondary only

Cluster 2

Typical of teachers from cluster 2 is the opinion that for students to be good in math, it is very important to understand concepts and principles, followed by the importance of being able to think in a sequential and procedural manner and to find reasons to support the solutions of problems.

Characteristics of Cluster 2		
Percentage of same answers	Variable	Variable value
88.95%	Btbmimp3: Teacher opinion: To be good in math - how important is it for students to understand concepts?	very important
84.34%	Btbmimp2: Teacher opinion: To be good in math - how important is it for students to think in a sequential and procedural manner?	very important

78.40%	Btbmimp6: Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	very important
78.15%	Btbgeduc: Teacher's highest level of education	BAor equivalent
75.76%	Btbmimp4: Teacher opinion: To be good in math - how important is it for students to be able to think creatively?	very important
75.76%	Btbmimp5: Teacher opinion: To be good in math - how important is for students to know how math is used in the real world?	very important
71.72%	Btbmwhr: How often do you record whether or not homework has been completed?	always
69.89%	Midea: In math lessons, how often a teacher uses a computer to show mathematic ideas?	never
68.89%	Musos: In math lessons, how often do students use an overhead projector?	never
68.40%	Musbt: In math lessons, how often do students use the board?	almost always
67.28%	Mcomp: In math lessons, how often do students use computers?	never
67.02%	Btbmwhr4: How often do you give feedback on homework in a class?	always
65.54%	Btbmles5: During lessons in mathematics, how often do students work in small groups without assistance?	some lessons

78% of teachers are highly educated, as they reached BA or equivalent. In large percentages they always record completion of homework and give feedback about it to their class. Only during a few lessons do students work in groups without teacher's assistance.
In average, almost 70% of students reported that teachers and students never use computers, and students also never use overhead projectors, but teachers almost always use boards.

Cluster 3

In cluster 3 teachers are quite different from the two preceding clusters and also from the others. Since students from this cluster achieved highest results, the table shows the whole set of characteristics.

Characteristics of Cluster 3		
Percentage of same answers	Variables	Variable value
87.31%	Btbmorar: How often do you have students prepare oral reports for homework?	never
78.85%	Btbgeduc: Teacher's highest level of education	BA or equivalen
76.68%	Musbt: In math lessons, how often does a teacher use the board?	almost always
67.48%	Btbmwrir: How often do you assign writing definitions, rules for homework?	never
66.54%	Btbmask2: How often do you ask students to use tables, charts or graphs to analyze relationships?	some lessons
63.63%	Btbmimp3: Teacher opinion: To be good in math - how important is it for students to understand concepts?	very important
61.94%	Btbgact5: Extracurricular time teacher spends to meet parents	less than 1 hour
60.53%	Btbmimp5:Teacher opinion: To be good in math - how important is it for students to know how math is used in the real world?	somewhat important
60.43%	Btbmfinr: How often do you have students find uses of the content for homework?	never
59.02%	Mhwfc: In math lessons, how often do students check each other's homework?	never
56.48%	Btdmcptm: Index of teacher's confidence to teach mathematics	high

56.11%	Btbmles5: In mathematics lessons how often do students work in pairs without assistance?	never
54.79%	Msmgp: In math lessons, how often do students work in small groups when beginning a new topic?	never
54.61%	Btbmdatr: How often do you assign small-scale research for homework?	never
53.10%	Btbmagr9: Teacher agreement with: A liking for and understanding of students are essential for teaching mathematics.	agree
52.35%	Btbgsex: Sex of teacher	male
51.97%	Btbmimp1: Teacher opinion: To be good in math - how important is it for students to remember formulas?	somewhat important
50.47%	Btbmimp4: Teacher opinion: To be good in math - how important is it for students to be able to think creatively?	somewhat important
50.28%	Btbmimp6: Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	very important

Teachers are most similar between themselves regarding the assignment of homework: as part of their homework assignment, 87% of teachers never ask students to prepare oral reports, 67% never ask students to write down definitions or rules, 60% never ask students to find uses of math contents or to make small-scale research.

The fact that 60% of teachers think it is only somewhat important for a student to be good in mathematics to understand how math is used in the real world, could be related to a characteristic of never asking students to find uses of math content.

Regarding cooperation between students, cluster 3 teachers also differ from other clusters. In average 60% of students reported they never correct each other’s homework, more than half of the students reported they never work in groups without teacher's assistance or begin learning a new topic by working in small groups. Besides, 62% of teachers see parents less often than one hour per week.

About a half of the teachers also think that for students to be good in mathematics, it is only somewhat important to think creatively or remember formulas, but it is very important for them that students are able to provide reasons to support their solutions.

67 % of teachers from this cluster are very self-assured in their ability to teach mathematics, half of them are men and also half of them agree (but not totally) that for students to be good in math, they need a teacher who likes them and understands their problems.

Cluster 4

Cluster 4 proved to be the group of teachers with the majority of answers missing.

Cluster 5

Characteristics of Cluster 5		
Percentage of same answers	Variables	Variable value
91.21%	Btbmask4: How often do you ask students to use computers?	never
83.20%	Btbmimp3 Teacher opinion: To be good in math - how important is it for students to understand concepts?	very important
80.04%	Mcomp: In math lessons, how often do students use computers?	never
79.78%	Btbmimp2 Teacher opinion: To be good in math - how important is it for students to think in a sequential and procedural manner?	very important
79.77%	Midea: In math lessons, how often teacher uses computer to show math ideas?	never

78.75%	Btbgeduc: Teacher's highest level of education	BA or equivalent
76.24%	Musos: In math lessons, how often do students use overhead projectors?	never
71.96%	Btbmimp4 Teacher opinion: To be good in math how - important is it for students to be able to think creatively?	very important
71.19%	Musbt: In math lessons, how often does a teacher use the board?	almost always
71.12%	Btbmimp6: Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	very important
67.70%	Musot: In math lessons, how often does a teacher use overhead projectors?	never

In math lessons of cluster 5 teachers, computers and overheads are typically not used by teachers. as well as by students. 91% of teachers and 80% of students said that students never use computers, 80% students report that teachers never use computers to show them math concepts. Over ¾ of students and more than 2/3 of teachers also never use overhead projectors.

Teachers primarily think that for students to be good in mathematics. it is very important to understand concepts (83.2%), to think in a sequential and procedural manner (79.8%), to be able to think creatively (72%) and to provide reasons to support solutions (71%).

Teachers completed high level of education, 78.8% of them reached BA or equivalent. Students report that large percentage of teachers (70%) almost always write on the board.

Cluster 6

A great majority of cluster 6 members come from Central and Eastern Europe. and have some very different characteristics from teachers in other clusters.

Characteristics of Cluster 6		
Percentage of same answers	Variables	Variable value
87.31%	Btbmorar: How often do you have students prepare oral reports for homework?	never
81.00%	Btbgsex: Sex of teacher	female
78.85%	Btbgeduc: Teacher's highest level of education	BA or equivalent
77.93%	Btbgsoap: Do you think that society appreciates your work?	no
76.68%	Musbs: In math lessons, how often do students use the board?	almost always
74.37%	Btbmimp6: Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	very important
74.21%	Btbmimp3: Teacher opinion: To be good in math - how important is it for students to understand concepts?	very important
72.76%	Btbmles6: In mathematics lessons how often do students work in pairs with assistance?	some lessons
72.27%	Btbmask3: How often do you ask students to work on problems with no obvious method of solution?	some lessons
72.19%	Btbmask2: How often do you ask students to use tables, charts or graphs to analyze relations?	some lessons
70.33%	Btbmimp4: Teacher opinion: To be good in math - how important is it for students to be able to think creatively?	very important
67.48%	Btbmwrir:How often do you assign writing definitions, rules for homework?	never
67.42%	Btbmles5: In mathematics lessons how often do students work in groups or pairs without assistance?	some lessons
66.54%	Btbmask2: How often do you ask students to use tables, charts or	some lessons

	graphs to analyze relations?	
64.59%	Mrule: In math lessons, how often does a teacher explain rules when introducing a new topic?	almost always

They are in 81% female, 78% of them have BA or have completed equivalent education, and 78% of them are convinced that the society doesn't appreciate their work in schools.

Regarding activities in mathematics classes, 87% of teachers never ask students to prepare oral reports as homework assignments. More than ¾ of cluster 6 teachers almost always use the board and around 2/3 of them almost always explain rules when they begin teaching a new topic.

A little less, around 72% of teachers, ask students to work on problems only during some of the lessons, for which there is no immediately obvious method of solution, ask them to analyze relations with the help of graphs and diagrams, or assist students when they work in groups. In average, 67% of students report about analyzing relations and working in groups without the assistance of a teacher only during some mathematics lessons. Because there were two more respondent categories, 'in most lessons' or 'every lesson' activities mentioned above don't seem to be very frequent.

These teachers think that for students to be good in mathematics, the most important is to provide reasons for supporting the solution of problems, followed by the importance of understanding mathematic concepts and principles.

Cluster 7

Teachers in cluster 7 are very similar between themselves in opinions that for students to be good at mathematics, it is very important to understand mathematic concepts and principles, which was reported by almost 90% of teachers. For students to be good at mathematics, to be able to provide reasons to support solutions of problems is also very important for almost 80% of teachers, and just slightly less teachers believe it is very important for students to be able to think in a sequential and procedural manner.

Characteristics of Cluster 7		
Percentage of same answers	Variables	Variable value
89.16%	Btbmimp3: Teacher opinion: To be good in math - how important is it for students to understand concepts?	very important
79.98%	Btbmimp6: Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	very important
77.58%	Btbmask2: How often do you ask students to use tables, charts or graphs to analyze relations?	some lessons
77.05%	Btbmimp2: Teacher opinion: To be good in math - how important is it for students to think in a sequential and procedural manner?	very important
72.83%	Musbt: In math lessons, how often does a teacher use the board?	almost always
72.39%	Midea: In math lessons, how often does a teacher use a computer to show math ideas?	never
71.26%	Btbmles6: In mathematics lessons how often do students work in pairs with assistance?	some lessons
70.81%	Btbmask3: How often do you ask students to work on problems with no obvious method of solution?	some lessons
70.05%	Btbmagr3: Teacher agreement with: Mathematics is a practical and structured guide for addressing real-life situations.	agree
69.15%	Btbgact5: Extracurricular time teacher spends to meet parents	less than 1 hour
69.15%	Btbmles5: In mathematics lessons how often do students work in pairs without assistance?	some lessons
69.07%	Btbgeduc: Teacher’s highest level of education	BA or equivalent

68.85%	Btbmwkbr: How often do you assign worksheets for homework?	sometimes
68.47%	Btbmwhr: How often do you record whether or not homework has been done?	always
68.02%	Btbmwhr7: How often do you use homework as a basis for class discussion?	sometimes
67.27%	Btbmask4 How often do you ask students to use computers?	never
67.11%	Mhwgv: In math lessons, how often does a teacher give homework?	almost always
66.44%	Btbmagr2: Teacher agreement with: Mathematics is primarily a formal way of representing the real world.	agree
66.21%	Musos: In math lessons, how often do students use overhead projectors?	never

More than ¾ of teachers ask students to analyze relations with the help of graphs and diagrams only during some of the lessons. Rarely, again in some lessons only, 71% of teachers ask students to work on problems without an obvious method of solution, or work in groups with or without assistance of a teacher.

Teachers from cluster 7 in 72% of cases almost always use the board and don't use a computer in mathematics lessons. More than 2/3 of students also reported that they've never used computers or overhead projectors by themselves during math lessons.

Regarding homework, for more than two thirds of the teachers, students report they almost always get homework, but only occasionally they are asked to do exercises from worksheets or workbooks. Teachers always record completion and sometimes use homework as a basis for discussion in a class.
Around 70% of teachers agree that mathematics is a practical and structured guide for addressing real situations and 66% just agree that mathematics is a formal representation of the real world. They could say that they totally agree.

An extended study of differences between clusters would require the observation of the whole distribution of variable values over clusters, using Methods of Categorical Data Analysis.

CONCLUSIONS

We tried to determine whether the ALCM is appropriate for analyzing data from international comparative studies in education. Our results show that the ALCM can be applied to a large amount of symbolic data, while its outcomes give a good basis for further analyses. In our case it detected significant differences among various groups of teachers which have not been observed as results of other methods.

Regarding our initial interest to see whether teaching mathematics is culturally independent, based on the result of the ALCM, the answer is clearly - no.

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Appendix A: Teacher Variables Included in Clustering Method

Teacher Variables	Descriptions of variables	Response categories
Btbgage	Age of teacher	1 = under 25, 2 = 25-29, 3 = 30-39, 4 = 40-49, 5 = 50-59, 6 = 60 / more
Btbgsex	Sex of teacher	1 = female, 2 = male
Btbgact1	Time teacher spends outside school to prepare test	1 = none, 2 = less than 1 hour, 3 = 1 - 2 hours, 4 = 3 - 4 hours, 5 = more than 4 hours
Btbgact2	Time teacher spends outside school to read student work	Same
Btbgact3	Time teacher spends outside school to plan lessons	Same
Btbgact4	Time teacher spends outside school to meet students	Same
Btbgact5	Time teacher spends outside school to meet parents	Same
Btbgact6	Time teacher spends outside school for professional reading	Same
Btbgmeet	How often a teacher has meetings with other teachers	1 = never, 2 = once or twice a year, 3 = every other month, 4 = once a month, 5 = once a week, 6 = two or three times a week, 7 = almost every day
Btbmimp1	Teacher opinion: To be good in math - how important is it for students to remember formulas?	1 = not important, 2 = somewhat important, 3 = very important
Btbmimp2	Teacher opinion: To be good in math - how important is it for students to think in a sequential and procedural manner?	Same
Btbmimp3	Teacher opinion: To be good in math - how important is it for students to understand concepts?	Same
Btbmimp4	Teacher opinion: To be good in math - how important is it for students to be able to think creatively?	Same
Btbmimp5	Teacher opinion: To be good in math - how important is it for students to know how math is used in the real world?	Same
Btbmimp6	Teacher opinion: To be good in math - how important is it for students to be able to support their solutions?	Same
Btbmagr1	Teacher agreement with: Mathematics is primarily an abstract subject.	1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree
Btbmagr2	Teacher agreement with: Mathematics is primarily a formal way of representing the real world.	Same
Btbmagr3	Teacher agreement with: Mathematics is a practical and structured guide for addressing real-life situations.	Same
Btbgagr4	Teacher agreement with: If students have difficulties, they should be given more practice by themselves.	Same
Btbgagr5	Some students have a natural talent for mathematics and others do not.	Same
Btbgagr6	Teacher agreement with: More than one representation should be used in teaching a mathematics topic.	Same
Btbmagr7	Teacher agreement with: Mathematics should be learned as a set of algorithms covering all possibilities.	Same
Btbmagr8	Teacher agreement with: Basic computational skills are sufficient for teaching primary school mathematics.	Same
Btbmagr9	Teacher agreement with: A liking for and understanding of students are essential for teaching mathematics.	Same
Btdmcptm	Index of teacher's self-confidence to teach mathematics	1 = low, 2 = medium, 3 = high
Btbgeduc	Teacher's highest level of education	1 = did not complete secondary school, 2 = secondary only, 3 = BA or equivalent, 4 = MA or PHD, 5 = nationally defined
Btbgsoap	Do you think that the society appreciates your work?	1 = yes, 2 = no
Btbmtxbr	Teacher's estimate of percentage of teaching time based on the textbook	1 = 0-25%, 2 = 26-50%, 3 = 51-75%, 4 = 76-100%
Btbmcal1	How often do your students use calculators for checking answers?	1 = almost every day, 2 = once or twice a week, 3 = once or twice a month, 4 = never, or hardly ever
Btbmcal2	How often do your students use calculators for tests and exams?	Same
Btbmcal3	How often do your students use calculators for routine computations?	Same
Btbmcal4	How often do your students use calculators for solving complex problems?	Same
Btbmcal5	How often do your students use calculators for exploring number concepts?	Same
Btbmask1	How often do you ask students to explain reasoning behind an idea?	1 = never or almost never, 2 = some lessons, 3 = most lessons, 4 = every lesson
Btbmask2	How often do you ask students to use tables, charts or graphs to analyze relationships?	Same
Btbmask3	How often do you ask students to work on problems with no obvious method of solution?	Same

Teacher Variables	Descriptions of variables	Response categories
Btbmask4	How often do you ask students to use computers?	Same
Btbmask5	How often do you ask students to write equations to represent relationships?	Same
Btbmask6	How often do you ask students to practice computational skills?	Same
Btbmles1	In mathematics lessons how often do students work individually without assistance?	Same
Btbmles2	In mathematics lessons how often do students work individually with assistance?	Same
Btbmles3	In mathematics lessons how often do students work as a class with teacher leading?	Same
Btbmles4	In mathematics lessons how often do students work as a class with students responding to each other?	Same
Btbmles5	In mathematics lessons how often do students work in pairs without assistance?	Same
Btbmles6	In mathematics lessons how often do students work in pairs with assistance?	Same
Btbmlm01	Is your teaching limited by students with different academic abilities?	1 = not at all, 2 = a little, 3 = quite a lot, 4 = a great deal
Btbmlm02	Is your teaching limited by students from a wide range of backgrounds?	Same
Btbmlm04	Is your teaching limited by uninterested students?	Same
Btbmlm05	Is your teaching limited by disruptive students?	Same
Btbmlm07	Is your teaching limited by parents uninterested in their children's progress?	Same
Btdmhwk	Amount of math homework assigned by a teacher per week	1 = never, 2 = < once/wk, >30 mins, 3 = < once/wk, <30 mins, 4 = 1-2/wk, >30 mins, 5 = 1-2/wk, <30 mins, 6 = 3 or more times/wk, >30 mins, 7 = 3 or more times/wk, <30 mins
Btbmwkbr	How often do you assign worksheets for homework?	1 = never, 2 = rarely, 3 = sometimes, 4 = always
Btbmpror	How often do you assign textbook problems for homework?	Same
Btbmrear	How often do you assign reading for homework?	Same
Btbmwrir	How often do you assign writing definitions, rules for homework?	Same
Btbmdatr	How often do you assign small investigations for homework?	Same
Btbmfinr	How often do you have students find uses of the content for homework?	Same
Btbmorar	How often do you have students prepare oral reports for homework?	Same
Btbmwhr	How often do you record whether or not homework was completed?	Same
Btbmwhr2	How often do you collect correct and keep homework assignments?	Same
Btbmwhr3	How often do you collect correct and return homework assignments?	Same
Btbmwhr4	How often do you give feedback on homework to whole class?	Same
Btbmwhr5	How often do you have students correct their own homework assignments in class?	Same
Btbmwhr6	How often do you have students exchange homework assignments and correct them?	Same
Btbmwhr7	How often do you use homework as a basis for class discussion?	Same
Btbmwhr8	How often do you use homework to contribute towards students' grades?	Same
Btbmwgt2	In assessment how much weight do you give teacher-made open-ended tests?	1 = none, 2 = a little, 3 = quite a lot, 4 = a great deal
Btbmwgt3	In assessment how much weight do you give teacher-made multiple-choice tests?	Same
Btbmwgt4	In assessment how much weight do you give homework assignments?	Same
Btbmwgt5	In assessment how much weight do you give project performance?	Same
Btbmwgt6	In assessment how much weight do you give observations of students?	Same
Btbmwgt7	In assessment how much weight do you give responses of students in class?	Same
Btbgass1	How often do you use assessment information to provide grades for students?	Same
Btbgass2	How often do you use assessment information to provide feedback to students?	Same
Btbgass3	How often do you use assessment information to diagnose learning problems?	Same
Btbgass4	How often do you use assessment information to report to parents?	Same
Btbgass6	How often do you use assessment information to plan for future lessons?	Same

Appendix B: Students Variables Included in Clustering Method

Students Variables	Description of variables	Response categories
Mday7	Outside school, how much time per week do you spend studying or doing homework in mathematics	1 = no time, 2 = less than 1 hour, 3 = 1-2 hours, 4 = 3-5 hours, 5 = more than 5 hours
Mcls1	Student's agreement with: In my math class, students often neglect school work	1 = strongly agree, 2 = agree, 3 = disagree, 4 = strongly disagree
Mcls2	Student's agreement with: In my math class, students are orderly and quite	Same
Mcls3	Student's agreement with: In my math class, students do exactly as the teacher says	Same
Mgood	Student's agreement with: I usually do well in math.	Same
Mlike	Student's agreement with: I like mathematics.	Same
Menjy	Student's agreement with: I enjoy learning math.	Same
Mbore	Student's agreement with: I think math is boring.	Same
Measy	Student's agreement with: I think math is an easy subject.	Same
Mprob	In math lessons, how often does a teacher show how to do problems?	1 = almost always, 2 = pretty often, 3 = once in a while, 4 = never
Mnote	In math lessons, how often do students copy notes from the board?	Same
Mtest	In math lessons, how often you have a quiz or test?	Same
Mproj	In math lessons, how often do students work on projects?	Same
Mwsht	In math lessons, how often do students work from worksheets on their own?	Same
Mcalc	In math lessons, how often do students use calculators?	Same
Mcomp	In math lessons, how often do students use computers?	Same
Mevlf	In math lessons, how often do students solve problems with everyday life things?	Same
Msgrp	In math lessons, how often do students work in pairs or small groups?	Same
Mhwgv	In math lessons, how often does a teacher give homework?	Same
Mhwcl	In math lessons, how often do students begin homework in class?	Same
Mhwtc	In math lessons, how often does a teacher check homework?	Same
Mhwfc	In math lessons, how often do students check each other's homework?	Same
Mhwds	In math lessons, how often do students discuss completed homework?	Same
Musbt	In math lessons, how often does a teacher use the board?	Same
Musot	In math lessons, how often a teacher uses an overhead projector?	Same
Musbs	In math lessons, how often do students use the board?	Same
Musos	In math lessons, how often do students use overhead projectors?	Same
Mrupt	In math lessons, how often does a teacher get interrupted?	Same
Midea	In math lessons, how often a teacher uses computer to show math ideas?	Same
Mrule	In math lessons, how often a teacher explains rules when introducing a new topic?	Same
Mprac	In math lessons, how often do students discuss practical problems when beginning a new topic?	Same
Msmgp	In math lessons, how often do students work in small groups when beginning a new topic?	Same
Mask	In math lessons, how often does a teacher ask what students know when beginning a new topic?	Same
Mtxbk	In math lessons, how often does a teacher ask students to look at the textbook when beginning a new topic?	Same
Meg	In math lessons, how often do students solve a related example when beginning a new topic?	Same
Mnbm02	National Mathematics benchmarks reached	1 = Did not reach the 25th percentile 2 = Reached the 25th percentile 3 = Reached the 50th percentile 4 = Reached the 75th percentile 5 = Reached the 90th percentile