THE DEVELOPMENT OF QUANTITATIVE REASONING AND QUANTITATIVE LITERACY

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Abstract

In this paper the secondary analysis carried out on the Israeli data collected in the framework of the Third International Mathematics Study (TIMSS 1999) is presented. In this study, the Israeli students were ranked in the 28th place out of a total of 38 countries, results which raised concern in the Israeli public. Following the international study, a number of research studies were carried out at the Ministry of Education, including the following: a secondary analysis of the international study, interviews with teachers, a comparative analysis between the Israeli curriculum and curricula in other countries, and an analysis of the changes that have taken place in the Israeli curriculum from the 1950s to the present. These studies indicate that one of the causes for the weak achievements of Israeli students is the lack of balance in the curriculum. The Israeli curriculum abandons the quantitative topics too soon in favor of formal and theoretical topics - algebra and deductive geometry. In this paper, the results of the research studies will be presented briefly, and the results of the secondary analysis of the International Mathematics Study will be presented at length. Following these research studies, the Ministry of Education launched a program "Developing Quantitative Reasoning." The program is accompanied by a controlled experiment. The interim results of the experiment show that we are on the right track.

BACKGROUND

Israel participated in the TIMSS 1999 study, in which 38 countries took part. The purpose of the test was to collect information on the students' achievements in mathematics and science and the factors affecting these achievements. The target population of the test was 13-year-old children (Mullis et al., 2000). In Israel, these are Grade 8 students. There are four stages in Israel's education system: Preschool (ages 3-6), elementary school (ages 6-12), lower secondary school (ages 12-15) and upper secondary school (15-18). This structure is the result of a gradual reform that the Israeli education system has undergone since the end of the 1960s. Today, 73% of Grade 8 students attend lower secondary schools, while the rest still attend eight-year elementary schools.

Israel was ranked in 28^{th} place in the 1999 test. The Israeli students' average score was 466 points, with a standard error of 3.9 - over five standard errors lower than average, in other words, significantly lower than the international average, which stood at 487 points.[1] As mentioned above, the test results raised great concern in Israel, and led to extensive discussions on the topic of education in general and education in mathematics and sciences in particular. The fact that Israel was ranked in first place in the international test held in 1964 (Husen, 1967) also contributed to the sense of distress in Israel with regard to the students' achievements. Following the results, two steps were undertaken to deal with the problem; one a long-term move aimed at a general reform of the education system, and the second a short and medium-term move focusing on topics needing improvement in mathematics education in Israel. This paper focuses on the move that deals with mathematics education in lower secondary schools.

In order to decide on the move that should be made, several research studies were carried out in the Ministry of Education. We will briefly present these studies:

1. **A secondary analysis of the international TIMSS 1999 study**

   An analysis of the test structure, teacher reports on the test and the students' achievements led to the conclusion that the quantitative area should be reinforced in lower secondary schools in Israel. This analysis will be presented below at length.

2. **Interviews with mathematics teachers**

   The main conclusion that arose from the interviews was that the quantitative topics are insufficiently studied in lower secondary schools; there is no review of quantitative topics studied in elementary school, and no enhancement and expansion of these topics. According to lower secondary school teachers, the inclusion of quantitative topics in the studies would change the results on international tests. In their opinion, devoting a relatively small amount of time to these topics would lead to a change in the students' achievements.

3. **Comparative analysis between the Israeli curriculum and curricula around the world**

   A comparison between the Israeli curriculum and curricula that include the quantitative topics indicates two types of differences with regard to including the quantitative topics in the curriculum: differences in the approach that are characteristic of the curriculum as a whole, and differences in content. With regard to differences in the approach, four areas were found that are characteristic of programs that include quantitative topics but are almost non-existent in Israel: continuity of learning, usage in everyday contexts, computations and limiting the use of calculators (in these curricula computations in general, and mental computations in particular, are perceived as central components in developing the students' quantitative ability), and estimation and rounding. As mentioned above, aside from differences in the approach, there are also differences in content, especially in the eras of Number sense, Measurements, Statistics and probability and Geometry. Moreover, in countries that include quantitative topics, many options are utilized for combining the topics of number sense, measurements, data analysis and probability for the purpose of mutual reinforcement. It can be concluded that the failure to deal with quantitative topics in Israel leads to a double loss - once due to the failure to teach these topics, and once due to the lack of mutual reinforcement.

4. Analysis of the changes in the Israeli curriculum from the 1950s to the present

In the middle of the 20th century, a number of historical events aroused a call in the United States to increase the number and quality of students pursuing higher scientific studies. This call led to a demand to teach theoretical and formalistic mathematics in schools, meaning that a greater emphasis was placed on teaching sets, structures and mathematical language. This move was later called "New Mathematics" (Senk & Thompson, 2003). It appears that the new approach also influenced the Israeli curriculum.

At the end of the 1960s, a profound change took place in the Israeli curriculum for lower secondary school, a change that stemmed in part from the processes mentioned above, but also from the structural modifications that occurred in the education system - the move from a three-stage structure (preschool, elementary school Grades 1-8 and secondary school Grades 9-12) to a four-stage structure (preschool, elementary school Grades 1-6, lower secondary school Grades 7-9 and upper secondary school Grades 10-12). The change in mathematics studies in lower secondary school was evident in the introduction of algebra studies starting in Grade 7 and Euclidean geometry starting in Grade 8, while neglecting the quantitative topics. Unlike the curriculum in effect in Israel today, until the 1970s substantial attention was paid to quantitative topics in Grades 7 and 8, particularly arithmetic problems involving computation and geometric measurements. The emphasis on quantitative topics was not limited to the study topics as these appeared in the curriculum, but also appeared in the guidelines that accompanied the learning, for example: a demand for mental computations, knowledge of facts and use of questions taken from everyday life. These gaps between the present Israeli curriculum and the curriculum used in the 1960s could provide an explanation, albeit a partial one, for the gaps between the achievements of students in the 1960s and the 1990s.

Following the studies presented above, we reached the conclusion that the balance in mathematics instruction in lower secondary schools was disrupted, with an inordinate emphasis placed on pure and formal mathematics while neglecting the quantitative topics. In light of this move, it was decided to initiate a move in Israel entitled "Developing quantitative reasoning and literacy and building a spiral learning method." This move is accompanied by a controlled experiment, and results from the first year of its operation indicate that we are on the right track.

THEORETICAL BACKGROUND

In recent years, we have witnessed growing interest in topics related to the quantitative aspect of mathematics studies; for example, the following terms can be found in the context of mathematics education: numeracy, quantitative literacy, quantitative skills, numeracy skills, numerical reasoning and quantitative reasoning.

The demand and need to deal with quantitativeness arises in many places in the literature, both due to the need for high quantitative ability for a reasonable level of functioning in modern society, and in order to reinforce the students' mathematical basis. As opposed to mathematics, quantitative ability does not continue to develop at an abstract level, but rather develops in more complex situations and contexts (The Quantitative Literacy Design Team, 2001; Orrill, 2001; Manster, 2001). Steen (1997) stated that despite years of learning and quantitative experience in life, educated people at times remain lacking quantitative ability. American students finish high school with quantitative skills that are inappropriate for life in modern society. It may be summarized that the need to teach the quantitative topics results from the desire and responsibility of education systems to provide an educated person with the ability to cope with the quantitative information that surrounds him to an increasing extent as technological progress and development move forward. In addition, there is a need to develop confidence in dealing with mathematics, and reduce anxiety of the subject, for both students and parents.

Numeracy appeared in the early 1980s and referred to an at-homeness with numbers and an ability to use mathematical skills in order to cope with daily demands. It also referred to the ability to evaluate and understand information that is presented in mathematical terms (Cockcroft, 1982). Many more definitions have developed since then, including that of NALS (1993) and IALS (1994) which stated that quantitativeness is about the knowledge and skills required to relate to single arithmetic operations or those within certain contexts, or as postulated by ILSS (2000), the knowledge and skills required to effectively cope with mathematical demands in various situations. The British curriculum for lower secondary school notes that: "numeracy is a proficiency that is developed mainly in mathematics but also in other subjects. It is more than an ability to do basic arithmetic. It involves developing confidence and competence with numbers and measures. It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts. Numeracy also demands understanding of the ways in which data are gathered by counting and measuring, and presented in graphs, diagrams, charts and tables" (The mathematics strand of the Key Stage 3, 2001).

In the context of the Israeli move, we have focused on two central components: developing quantitative reasoning and improving the students' ability to deal with everyday mathematical problems.

Developing quantitative reasoning - the ability for quantitative reasoning refers to logical ability and systematic thinking, including the use of skills such as raising hypotheses, analysis, estimation, generalization, merging, solving unconventional problems and proving or disproving arguments. Reasoning is considering part of the higher cognitive processes associated with the study of mathematics. In order for students to be able to engage in these activities in quantitative topics in lower secondary school, a spiral learning model must be developed, which includes a review of the material studied in elementary school, as well as additions to what was studied in the elementary framework and expansion of these studies.

Improving the students' activity to deal with everyday mathematical problems (quantitative literacy) - teaching quantitative topics taken from the world in which we live, in order to turn the students into people who can function with the quantitative information that surrounds them today and in the future.

SECONDARY ANALYSIS

The analysis of the study is divided into two parts:

- An analysis of findings that appear in the international report (Mulls et al., 2000) and in the national report (Zuzovsky, 2001).
- An Items analysis, using material published by the IEA: Percent of Responses by Item Category for the Mathematics Item Main Survey, TIMSS 1999 Database and TIMSS 1999 User Guide.

What is learnt from the national report and the international report?

1. Gap between the intended and implemented curriculum in Israel and the intended and implemented curricula in countries that took part in the TIMSS 1999 study.

An examination of the content of the international study and the curriculum in Israel points at a gap. The curriculum in the study includes five content areas: fractions and number sense (numbers); algebra; geometry; measurements; and data representation, analysis and probability. The Israeli curriculum concentrates on two topics: algebra and geometry. It is important to emphasize the gap, since this gap in itself points at a fundamental difference between Israel and the majority of the countries that participated in the study. Indeed, one of the criteria for including a topic in the test was that at least two-thirds of the countries studied the topic. Statistics and probability appear in the Israeli curriculum (under the topic of algebra), but are not given great emphasis either in the intended curriculum or in the implemented curriculum. And as we will see below, it should be added that the geometry area that appears in the study is fundamentally different from the area studied in Israel. More details about these gaps can be obtained from a number of exhibits in the international report.

The following table describes the teachers' responses to the question "What mathematics content do teachers emphasize in Grade 8?"

T*able 1: Percentage of Students Whose Teachers Report the Subject Matter Emphasized Most in Their Grade 8 Mathematics Class*[2]

	Mainly Number	*Combined Algebra, Geometry, Number,etc.*	*Combined Algebra and Geometry*	*Algebra*	*Geometry*	*Other*
Israel	1 (0.4)	35 (4.0)	42 (4.1)	19 (3.4)	1 (0.6)	2 (1.3)
International Avg.	14 (0.4)	55 (0.6)	19 (0.5)	8 (0.4)	3 (0.2)	2 (0.2)

This table shows that in most countries, algebra, geometry, numbers and other topics in mathematics are taught, whereas in Israel algebra and geometry are the focus of teaching. The report indicates that over 60% of students in Israel study with teachers who concentrate on algebra and/or geometry, whereas the international average is 30%. Teachers of 36% of students in Israel report that they combine quantitative topics in the studies, as opposed to 69% around the world. This gap is the result of a gap that exists in both the intended and implemented curricula. In addition, geometry studies in Israel focus on the process of proof and understanding an axiomatic system - topics which do not appear in the international study. This gap in geometry studies can be seen, *inter alia*, in the Israeli national report.[3] Out of 22 items belonging to the area of geometry, three items belong to algebra studies (coordinates), and out of the remaining 19 items, 11 were not studied. In other words, 64% of these items are not studied as part of geometry studies in Israel.

More evidence of the failure to deal with quantitative topics in mathematics studies in Israel can be found in the responses to the questions that appeared in the teachers' questionnaire, Section B (Questions 13a-Questions 13f). The teachers were requested to state whether the students in their class had studied each of the topics. The responses to these questions do not clarify the amount of time devoted to studying the topic, but can provide a good indication as to whether the topic was studied during the present school year.

Table 2: Percentage of Students who have Studied Different Study Topics[4]

		Taught topics before this year only	*Taught topics during this year*		*Not yet taught 50% or more of topics*
			More than 50% of topics taught	*50% or less of topics taught*	
Fractions and Number Sense	Israel	75	21	4	0
	International Avg.	50	45	4	1
Measurements	Israel	51	13	7	29
	International Avg.	45	40	8	6
Data and Probability	Israel	25	18	13	44
	International Avg.	19	39	7	34
Geometry	Israel	2	31	20	47
	International Avg.	16	42	20	22
Algebra	Israel	12	77	9	1
	International Avg.	15	73	4	8

This table indicates that in Israel, only 25% of the students studied fractions and number sense during the school year when the study was held, while the international average is nearly double - 49%. In the area of measurements the picture is even more severe, with only 20% of students in Israel studying the topic during the year, while the international average is nearly two and a half times this figure - 48%. Moreover, if out of the students who studied the topic we separate the students who studied more than half of the topics appearing in the area of measurements during the year, we will find that the gap is even larger - in Israel, 13% of students studied more than half of the topics, while the international average is 40%, i.e., over three times as high. In data representation and probability, 31% of students in Israel studied the topic during the year, while the international average is 46%. Here too, when considering those students who studied more than half of the topics, the gap grows. In geometry there is a smaller gap, but as mentioned earlier, it is hard to learn from this figure, due to the difference between the geometry curriculum in Israel and the study's geometry curriculum. The Israeli curriculum concentrates on familiarity with the axiomatic structure and basic concepts, and proving theorems and exercises - areas that do not appear in the study. In algebra, it can be seen that the picture is reversed - this is the only subject studied by a higher average proportion of students in Israel during Grade 8, in comparison with the international average. These findings are compatible with other research studies carried out, which indicated that the study of mathematics in lower secondary schools in Israel is not balanced in comparison with other schools around the world.

2. Use of calculators in learning

A number of mathematics experts, such as Klein and Milgram (1999) assert that a failure to deal with traditional algorithms will eventually undermine basic skills. Andersen (1998) argued that excessive use of calculators could also undermine basic skills. In Israel there are no guidelines in the curriculum regarding calculators and the extent of their use, nor is there a requirement to carry out calculations with paper and pencil or mental computations. In many curricula around the world, an emphasis is placed on this matter at the lower secondary school stage. For example, the British curriculum reads: "An ability to calculate mentally lies at the heart of numeracy. As a teacher, whether of mathematics or another subject, you should stress the importance of mental computation methods and give all pupils regular opportunities to develop the skills involved."[5]

The following table details the responses of teachers to two questions that appeared on the teachers' questionnaire - Question 5, which inquired whether the students have access to calculators in class, and Question 6, which inquired into the policy on use of calculators.

Table 3: Calculator Use in Mathematics Class[6]

	Percentage of students having access to calculators in class	Policy on use of calculators during mathematics lessons for students having access		
		Unrestricted use	Restricted use	Calculators not permitted
Israel	98 (0.8)	78 (3.0)	21 (3.0)	1 (0.1)
England	100 (0.3)	14 (2.2)	86 (2.2)	0 (0.0)
International Avg.	73 (0.5)	21 (0.5)	67 (0.7)	12 (0.6)

This table reinforces the argument regarding the unrestricted use of calculators in mathematics classes in Israel. It can be gathered from the table that 98% of students in Israel have access to calculators in class, as opposed to an international average of 73%. This difference does not necessarily stem from different approaches, and can also be due to other reasons (e.g., economic reasons), as evidenced by the average in England (100%). Conversely, in Israel most of the students study with teachers who do not restrict the use of calculators in mathematics classes - 78%. The international average of students whose teachers do not restrict the use of calculators is 21%, in other words, about a quarter of the average number in Israel. In England, the average proportion of students whose teachers do not restrict the use of calculators is 14%, in other words, less than a fifth of the Israeli average. These differences can indicate the existence of different approaches in this area.

3. Student achievements in various content areas

The international report shows that only in algebra, the achievements of the Israeli students did not fall significantly below the international average, while in the other subjects the average achievements of Israeli students were significantly lower than the international average.

The low score in geometry can be explained, as mentioned above, by the large difference between the study topics, their goal and instruction methods. In addition, the instruction of deductive Euclidean geometry is apparently appropriate for a limited number of students. This topic cannot be checked by means of the international study, since questions that examine axiomatic understanding and the ability to formulate a geometric proof did not appear in the study.

It appears that the failure to deal with quantitative topics in lower secondary schools in Israel, and the failure to implement a spiral learning method, which expresses a rift between studies in elementary school and lower secondary school, affect the students' achievements in the quantitative topics, and may also lower the students' achievements on other topics.

Table 4: Average Achievements of Israeli Students in Different Content Areas - TIMSS, 1999[7]

Content Area	*Israeli Average*	*International Average*	*Difference from Average*	*Statistical Significance*
Fractions and Number Sense	472 (4.4)	487	15	Significantly lower than international average
Measurements	457 (5.1)	487	30	Significantly lower than international average
Data Representation and Probability	468 (5.1)	487	19	Significantly lower than international average
Geometry	462 (5.4)	487	25	Significantly lower than international average
Algebra	479 (4.5)	487	8	Not significantly different from international average

What is learnt from the analysis of the test items?

As noted above, the failure to include quantitative topics can have many effects on mathematics studies. For example, an unsteady basis in mathematics can cause a lack of confidence in dealing with the subject, and a weak computation ability can also lower the students' achievements in algebraic problems. In addition, a failure to deal with arithmetic problems does not enable students to find an arithmetic solution to an algebraic problem, when sometimes the arithmetic solution is more natural or simpler. A failure to deal with mental computations can reduce the understanding of number sense, leading to problems in representing fractions. Since the effect of the failure to include quantitative topics is broad, and the students' achievements in quantitative topics were low, it is not an easy task to find a broad group of items that matches the international classification of items and highlights the effect of the failure to deal with quantitative topics. However, we estimated that if we isolated from the areas of number sense and measurements (content areas that are almost not studied at lower secondary schools in Israel) the topics that require computation ability, the achievements of the Israelis students would be lower as compared to the achievements in the area being tested (number sense or measurements). We decided to choose the topic of computation questions that appears in the area of fractions and number sense, and the topic of areas and perimeters that appears in the area of measurements. There are several reasons for this choice:

1. We estimated that the failure to study these topics, in addition to the lack of restrictions on use of calculators, has an effect on the students' achievements in these topics.
2. Countries that include quantitative topics place an emphasis on the topic of problems and arithmetic calculations, and on the topic of measuring perimeters, areas and volumes.
3. These topics were of particular interest to us since they were topics where the greatest changes had taken place, in a comparison between the Israeli curriculum employed in the 1960s and the present curriculum.

We wished to use these topics to examine the hypothesis that the failure to include quantitative topics in the curriculum lowers student achievement. Conversely, since algebra is an area most studied in Israel, the hypothesis was that if we separate the study items in algebra covered in the Israeli curriculum, the average achievements of Israeli students will rise.

ANALYSIS

An initial analysis examined, in a descriptive manner, whether there is a difference between the average achievements in the topic being examined, and the average achievements in the content area to which the topic belongs. The difference was checked both on a national and an international level. We carried out a simple arithmetic average of the percentage of correct answers in each area.

When a comparison is made of the average achievements of students and the average of all participating countries, differences may be found that result from differences between student levels, and this can be misleading. Therefore, researchers suggest making comparisons on specific topics between countries with no significant differences in the average student achievements between them (Routitsky & Zammit, 2002; Postlethwaite, 1999). For this reason, comparisons were also made between the average achievements of Israeli students in the topics being examined, and countries that do not have significant differences in achievements as compared to Israel. In addition, a comparative analysis was carried out among students in Israel between the achievements in the topic being examined and the other questions in the content area. For the purpose of carrying out the analysis, the respondents were divided in quartiles, in accordance with the Israeli average. The analysis was carried out at the quartile level, to see whether the phenomena that were found exist for students at different levels. The dependent variable, the percentage of correct answers, was calculated by examining the percentage of correct answers out of the total number of answers received by the student in this area, and multiplying them by the House Weight, which refers to the complex sample taken in each country, as suggested by the IEA (TIMSS

1999, User Guide, 5-9). The adjustment of these weights changed the range of the dependent variable, so that it extended from 0-269 for the number area, 0-403 for the measurements area, and 0-239 for the algebra area.

Since in some of the tests there was a high percentage of respondents who did not answer questions, in order to preserve the normal distribution the analyses were carried out through a logarithmic transformation on the dependent variables. However, the averages presented are the pre-transformation averages, in order to enable easier interpretation.

In order to examine the differences between quartiles, an ANOVA for Repeated Measures 4x2 test was carried out, with the independent variables being the quartile to which the respondent belongs and the type of questions, and the dependent variable being the percentage of correct answers. Further analyses for examining the source of statistical significance were carried out using Bonferroni corrections on the average. The results of the analyses are detailed in the tables below.

1. Fractions and Number Sense (Numbers)

As we have stated, in the area of numbers the main difference between the Israeli curriculum and curricula around the world is the failure to deal with arithmetic in lower secondary school, and mainly the lack of attention to arithmetic problems and arithmetic calculations. The lack of restrictions on use of calculators and the lack of demand for mental computations as well as paper and pencil computations heighten the problem. Since the use of calculators was not allowed in the 1999 test, the hypothesis was raised that within the area of numbers, if we isolate the questions related to computations we will find a drop in student achievements. We will mark the group of questions in the area of numbers as A. This group has 61 items, and according to the international classification this area is composed of the following subtopics:

Table 5: Breakdown of the 61 Items in the Fractions and Number Sense Test (Group A), according to the International Classification

Topic	*No. of items*	*Topic*	*No. of items*
Computations with whole numbers	2	Relationships between common and decimal fractions	3
Common fractions	12	Rounding whole numbers and fractions	5
Computations of common fractions	9	Estimating the results of computations	6
Decimal fractions	3	Percentages	2
Computations with decimal fractions	9	Ratio and proportion	9

In order to test the hypothesis, we chose the two large groups that deal with the topic of computations: Computations with common fractions and computations with decimal fractions, numbering 18 items in total. This group will hereinafter be called the group of "computations with fractions." We will mark this group as B. Within the items belonging to the group, arithmetic computation problems can be found, such as:

R14: Leah had NIS 240 and used $\frac{5}{8}$ of the money. How much money did she have left?

N17: A painter had 25 liters of paints. He used 2.5 liters every hour, and finished his work 5.5 hours later. How much paint did he have left?

Or computational exercises: L18: $\frac{4}{5} - \frac{1}{3} - \frac{1}{15} =$ or R7: Calculate: 4.722-1.945 =

Out of the 18 items in Group B, seven are computational exercises like the last two items, and the others are computational problems like the first two items. The seven computational exercises will be hereinafter called the group of computational exercises. We will mark this group as C.

Below is a table comparing the average percentage of correct answers (simple arithmetic average) in Group A, Group B and Group C.

Table 6: Average of Correct Answers in the Area of Fractions and Number Sense

	A: Area of numbers		*B: Computations of fractions*		*C: Computational exercises*		*d1*	*d2*
	Avg.	*N*	*Avg.*	*N*	*Avg.*	*N*		
No. of items	61		18		7			
Simple Avg. in Israel	46.4	4,190	36.8	4,190	35.2	4,190	9.6	11.2
Simple Avg. - Comparison countries	48.4	39,050	44.6	39,050	47.4	39,050	4.2	1.4
Simple international Avg.	52.7	180,300	48.4	180,300	50.5	180,300	4.3	2.2

Notes:

- d1 is the difference between the average in Groups A and B, and d2 is the difference between the average in Groups A and C.
- The comparison countries are: Cyprus, Israel, Italy, Lithuania, Moldova, New Zealand, Romania and Thailand. There was no statistically significant difference between these countries in the average achievements of students in the area of numbers.[8]

The table shows that in Israel, there is a large drop of close to ten percentage points in the topic of computations with common and decimal fractions, in comparison with the entire area of numbers. The drop in the average percentage of correct answers is larger than the drop both in the comparison countries and in the international average. In addition, in the area of computational exercises, the gap in the average percentage of correct answers grew over 11 percentage points while in both the comparison countries and the international average the gap shrank to an average of 1.4 and 2.2 percentage points.

Among the comparison countries there is one that is similar to Israel: in New Zealand, the average percentage of correct responses in the area of numbers is 50%, 37.1% in computations with fractions, and 27.2% in computational exercises; in other words, the gaps in New Zealand are even greater than those in Israel. It is interesting to note that among the comparison countries, Israel and New Zealand are the only ones where the teachers of more than 50% of the pupils stated that they do not restrict the use of calculators during math lessons (78% of pupils in Israel, 60% of pupils in New Zealand).

In the remaining countries, the percentage of pupils whose teachers reported that they do not restrict the usage of calculators in class ranges from 4% in Romania to 28% in Moldova.

Additional analysis examined whether the decline is present in all of the quartiles. Two comparative analyses were carried out between the computation with fractions (Group B - 18 questions) and the group dealing with remaining items in the area of numbers (43 questions), hereinafter, Group E (E = A – B). There was also a comparison between the Computational Exercises group (Group C - 7 items) and the remaining items in the area of numbers (54 questions), hereinafter, Group F (F = A – C).

Table 7: Average, Standard Error and Differences Between the Percentage of Correct Answers in Groups E and B, by Quartiles (N = 4180)

Quartiles	*E: Average, of remaining items*	*B: Average, computations with fractions*	*d*	*N*
Number of items	43	18		
1st quartile (lower 25%)	29.79 (24.44)	19.05 (36.29)	10.74	1039
2nd quartile	40.18 (18.73)	24.82 (34.35)	15.36	1045
3rd quartile	55.67 (20.90)	39.01 (41.16)	16.66	1048
4th quartile (upper 25%)	76.92 (24.08)	61.48 (42.07)	15.44	1048

Results of the difference test showed that there is a primary effect regarding the "type of question" variable ($F(1,4176) = 2499.10$; $p < 0.01$; $n = .37$), so that in all of the quartiles we can see that the percentage of correct answers in computational problems is lower than the percentage of answers in the other areas of numbers. Furthermore, a primary effect was found regarding the "quartile" variable ($F(3,4176) = 444.12$;

p < 0.01; n = .24). Later analysis conducted using the Bonferroni correction showed that each quartile was significantly higher than the previous quartile. In addition, an interaction was found between the type of answers and the quartiles (F (3,4176) = 42.73; p < 0.01; n = .03). From Table 7 we can see a tendency according to which the smallest difference was found in the first quartile, compared with the other quartiles. Moreover, the extent of the effect (n) indicates that the type of questions and the quartiles contribute much to explaining the difference in the percentage of correct answers while the contribution of the interaction is less significant.

Table 8: Average, Standard Deviation and Differences Between the Percentage of Correct Answers in Groups F and C, by Quartiles (N = 2600)

Quartiles	*F: Average, of remaining items*	*C: Average, computational exercises*	*D*	*N*
Number of items	54	7		
1st quartile (lower 25%)	28.34 (20.25)	18.44 (41.43)	9.90	621
2nd quartile	40.81 (18.58)	24.22 (40.22)	16.59	658
3rd quartile	55.85 (20.53)	40.23 (44.95)	15.62	670
4th quartile (upper 25%)	76.22 (23.96)	60.34 (46.80)	15.88	651

Results of the test indicated that there is a primary effect regarding the "type of question" variable (F (1,2596) = 2116.61; p < 0.01; n = .45), so that in all of the quartiles we can see that the percentage of correct answers in computational exercises is lower than the percentage of answers in the other areas of numbers. Furthermore, a primary effect was found regarding the "quartile" variable (F (3,2596) = 114.99; p < 0.01; n = .24). Later analysis conducted using the Bonferroni correction showed that each quartile was significantly higher than the previous quartile. In addition, an interaction was found between the type of answers and the quartiles (F (3,2596) = 34.75; p < 0.01; n=.04). From Table 8 we can see a tendency according to which the smallest difference was found in the first quartile, compared with the other quartiles. Moreover, the extent of the effect (n) indicates that the type of questions and the quartiles contribute much towards explaining the difference in the percentage of correct answers while the contribution of the interaction is less significant.

Discussion: The achievements of Israeli pupils in the area of numbers are significantly lower than the international average. In the area of numbers we found a partial topic in which Israeli pupils were particularly weak. This partial area deals with the subject of computations with fractions. There is a drop of 9.6 percentage points in the average score with regard to fraction computation, compared with the average score in the area of numbers. When we limit the topic of the computational exercises (Group C), the gap increases to 11.2 percentage points. In the comparison countries where the average score was similar to that of Israel, the score in computational exercises drops by only 1.4 percentage points compared with the average score in the area of numbers. A similar discrepancy also exists in the international average. It would appear that the fact that the subject of numbers is taught so little in Israeli lower secondary schools and unrestricted use of calculators has an impact on achievements, particularly with regard to the topic of computations with fractions. The decline in achievements in computations using fractions is present in all quartiles. Analysis shows that in all of the quartiles there is a statistically significant gap between pupil achievements in the group with questions on computations with fractions (Group B) and the group with the remaining questions (Group E). Similarly, there is a discrepancy between the achievements of the pupils on computational exercises (Group C) and the supplementary group (Group F). In other words, "particularly low" achievements in computations with fractions may be found among all levels of the student population.

One of the reasons that Israel stopped teaching fractions in the lower secondary schools was the assumption that further occupation with fractions would not help those pupils who had been unsuccessful with the subject even in elementary school. Either the pupil understood the subject or he would never understand it. The analysis shows that when the topic of numbers is not studied in lower secondary school, even pupils at higher levels have particular trouble coping with the subject of computations with fractions. The average score in the 4^{th} quartile (upper 25%) for computation exercises with fractions was 60 versus a score of 76 in other subjects in the area of numbers. In the 3^{rd} quartile the gap was between a score of 40 on computation exercises with fractions and 56 on other subjects in the area of numbers. An additional result of the fact that there is no arithmetic practice in the lower secondary schools is that methods for solving problems are fixed. To illustrate this, below is exercise TO1 (from the algebra area).

TO1: A club has 86 members and there are 14 more girls than there are boys. How many boys and how many girls are there in the club?

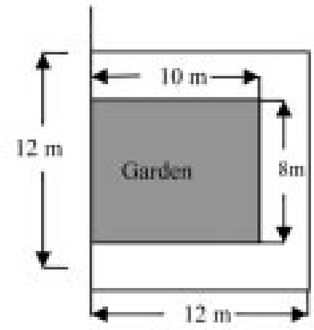
There are two main methods that can be used to solve this problem:

A. Arithmetic – $86 : 2 = 43 \Rightarrow 43 - 7 = 36$ boys $\Rightarrow 43 + 7 = 50$ girls.

B. Algebra – $X + (X + 14) = 86 \Rightarrow 2X = 72 \Rightarrow X = 36$ boys $\Rightarrow X + 14 = 50$ girls.

In Israel, the percentage of pupils who solved the problem using algebra (18.3%) is almost double the percentage of pupils who solved the problem using arithmetic (10.9%). The international average shows that more pupils on average solve this problem using arithmetic than using algebra. Apparently, the inclusion of arithmetic problems in the curriculum and restricting the use of calculators can improve pupil achievements in the area of numbers in general, and regarding computations with fractions in particular, and can also encourage pupils to solve problems using a variety of methods.

A rectangular plot adjacent to a building has a path around the other three sides, as shown. What is the area of the path?



2. Measurements

As we have indicated, another change in the Israeli curriculum that began in the 1970s was that pupils were not required to deal with the topic of measurements, particularly determining area and volume. Since a great proportion of the questions in this area require computation, the theory was raised that Israeli pupils would find this subject especially difficult as well. Take, for example, exercise J10, for which the achievements of the Israeli pupils were significantly below the international average – 28% answered this correctly (standard error of 2.1), while the international average was 42% (standard error of 0.4). In this exercise the pupils had to perform calculations. There are several possible methods for solving the problem, one involving the direct calculation of the area of the path, that is: $2 \cdot (10 \cdot 2) + 12 \cdot 2 = 40 + 24 = 64$. We could use another method that takes the whole and deducts the garden's part to find the area we want to know, i.e., $12 \cdot 12 - 10 \cdot 8 = 144 - 80 = 64$. The options for performing calculations, and in many cases to distinguish between options for direct or indirect computation characterizes many problems in this topic, as well as a verity of arithmetic problems (see, for example, questions R14 and N17, above). Not studying computational arithmetic problems and, problems involving area and volume can jeopardize the pupils' ability in computational skills, as well as their ability to choose computational strategies.

The area of measurement, which is indicated below as H, is composed of six subtopics, as follows:

Table 9: Distribution of 24 Items in the area of Measurement, According to the International Classification

Topic	*No. of items*	*Topic*	*No. of items*
Units of measurement - standard metric units	5	Perimeter and area of simple shapes	6
Reading measurement instruments	2	Perimeter and area of combined shapes	6
Estimated of measurement	4	Volume of rectangular solids	1

We focused on a group that included the following subjects: Perimeter and area of simple shapes, perimeter and area of combined shapes, and volume (13 questions). Hereinafter, this group will be known as the Area and Perimeter Group, and will be indicated by the letter G. We also used two methods: one based on average pupil achievements on the items, and the other using a difference test for repeated measurements.

Table 10: Average Percentage of Correct Answers in the Area of Measurements (Group H) and on the topic of Areas and Perimeters (Group G), in Israel, in the Comparison Countries, and International Average

	Measurement Area (H)		*Areas and Perimeters (G)*		*d*
	Average	*N*	*Average*	*N*	
Number of items	24		13		
Simple average in Israel	36.5	4,190	22.8	4,190	13.7
Simple comparative average	37.1	39,050	27.5	39,050	9.7
Simple international average	46.4	180,300	37.0	180,300	9.4

Note: Comparative countries are: Cyprus, Israel, Jordan, Lithunia, Macedonia, Moldova, Thailand, Tunisia and Turkey[9].

The table shows that the subject of areas and perimeters was also difficult at the international level, while the analysis indicates that the decline in achievement in Israel was greater in relation to the decline in the international average and with the comparison countries.

Table 11: Average, Standard Deviation and Differences Between the Percentages of Correct Answers in the Topic of Areas and Perimeters (Group G) and the Remaining Questions in the Area of Measurements (Group H-G), by Quartiles (N = 3659)

Quartiles	*Group of items that completes G (H – G)*	*Areas and Perimeters (Group G)*	*d*	*N*
Number of items	11	13		
1st quartile (upper 25%)	25.92 (36.49)	13.17 (34.40)	12.75	915
2nd quartile	42.38 (31.01)	15.33 (29.56)	27.05	930
3rd quartile	62.00 (34.46)	26.17 (36.24)	35.83	906
4th quartile (bottom 25%)	78.93 (31.54)	52.98 (43.13)	25.95	908

Results of the test showed that there is a primary effect regarding the "type of question" variable ($F(1,3655)=1858.95$; $p<0.01$; $n=.34$), so that in all of the quartiles we can see that the percentage of correct answers in problems with area and perimeter is lower than for the remaining questions in the topic of measurements. Furthermore, a primary effect was found regarding the "quartile" variable ($F(3,3655)=586.86$; $p<0.01$; $n=.32$). Later analysis conducted using the Bonferroni correction showed that each quartile was significantly higher than the previous quartile. In addition, an interaction was found between the type of answers and the quartiles ($F(3,3655)=35.13$; $p<0.01$; $n=.03$). From Table 11 we can see a tendency according to which the smallest difference was found in the first quartile, compared with the other quartiles.

Discussion: The achievements of the Israeli pupils in the area of measurements are significantly lower than the international average. Here, too, we found a partial area that included perimeters, areas and volumes, in which there was a decrease of some 30 percentage points in each quartile (except for the lowest quartile), compared with the average score that included all the remaining questions in the area of measurements (H-G). The fact that the topic of perimeter, area and volume is not covered in Israeli mathematics studies, and the need to perform computations and choose computational strategies as illustrated above, raises the hypothesis that inclusion of this subject in the curriculum would improve student achievements. Moreover, dealing with the area of numbers and measurements in general, and subjects of calculations with fractions and area and perimeter in particular, could promote mutual reinforcement and enhancement between these two areas; this indicates a dual loss that derives from the fact that neither of the areas mentioned are taught, and because there is no mutual reinforcement between them.

3. Algebra

To complete the analysis we wanted to examine the subject of algebra, the only area in Israel that is not significantly below the international average. Details of the test in algebra were divided into six subtopics, as follows:

Table 12: Distribution of 35 Test Items in Algebra, According to International Classification

Topic	*No. of items*	*Topic*	*No. of items*
Number patterns and simple relations	6	Solving simple equations	5
Simple algebraic expressions	5	Solving simple inequalities	1
Representing situations algebraically; formulas	15	Ratio and proportion	3

In Israel, the subject of algebra focuses on the following areas: simple algebraic expressions, representing situations algebraically, formulas solving simple equations and solving simple inequalities. This group of exercises was combined and called learned algebra. Out of the group of items - representing situations algebraically, formulas formally, exercises S1 and V1 were removed as they are not part of the Israeli curriculum (finding sequences and extrapolation).

Table 13: Average Percentage of Correct Answers in the Area of Algebra and Learned Algebra, in Israel, in Comparison Countries and International Averages

	Areas of Algebra		*Learned Aldgebra*		*d*
	Average	*N*	*Average*	*N*	
Number of items	35		20		
Simple average in Israel	46.3	4,190	50.3	4,190	4.0
Simple comparative average	46.2	39,050	48.7	39,050	2.5
Simple international average	49.0	180,300	50.8	180,300	1.8

Note: Comparative countries are: Cyprus, England, Israel, Italy, Latvia, Lithuania, Macedonia, Moldova, New Zealand and Romania[10].

The analysis shows that in the area of Learned Algebra, in subjects that were taught, the Israeli average compared with the average for Algebra improved by more than 4 percentage points. This improvement is greater than the improvement in both the comparison countries and in comparison with the international average. A similar phenomenon can also be seen when analyzing the quartiles, as demonstrated by the following table.

Table 14: Average, Standard Deviation and Differences Between the Percentages of Correct Answers in the Area of Learned Algebra and the Remaining Items in the Area of Algebra, by Quartiles (N = 3659)

Quartiles	*Learned Algebra that completes*	*Remaining items*	*d*	*N*
Number of items	20	15		
1st quartile (lower 25%)	26.40 (37.53)	22.48 (30.64)	3.92	1046
2nd quartile	43.40 (25.21)	35.03 (34.56)	8.37	1047
3rd quartile	61.58 (26.23)	54.86 (37.57)	6.72	1048
4th quartile (upper 25%)	81.98 (28.06)	78.10 (33.68)	3.88	1048

Results of the test showed that there is a primary effect regarding the "type of question" variable ($F(1, 4185) = 413.35$; $p < 0.01$; $n = .09$), so that in all of the quartiles we can see that the percentage of correct answers in algebra problems that were not studied is lower than the percentage of algebra questions that were studied. Furthermore, a primary effect was found regarding the "quartile" variable ($F(3,4185) = 943.83$; $p < 0.01$; $n = .40$). Later analysis conducted using the Bonferroni correction showed that each quartile was significantly higher than the previous quartile. In addition, an interaction was found between the type of answers and the quartiles ($F(3,3655) = 30.86$; $p < 0.05$; $n = .02$).

Concluding Discussion: The results show that pupil achievements in Algebra topics that were not studied are lower than Algebra topics that were studied. The decline in achievement can be found in all quartiles. The drop in the fourth quartile (3.9 percentage points) is lower than the drop in the third and second quartiles (6.7 and 8.4 percentage points, respectively). One possible explanation for this is that in Algebra, the better pupils (fourth quartile) are better at coping with topics that weren't taught formally. Attention should be paid to the descending order of the discrepancy in achievement, from the fourth quartile to the second quartile. This phenomenon was not noticed with regard to fractions and measurements. There, no difference or trend was noticed when analyzing the decrease in the three highest quartiles (see Tables 7, 8, 11). These results reinforce the claim that when calculations are not studied together with fractions, and area, perimeter and volume, the negative influence is felt similarly at all levels above the bottom 25%. In other words, as opposed to algebra, in the areas of numbers and measurements that were tested, even the good pupils "suffer." This can also be supported by the extent of the effect (n) such that with regard to Algebra, the type of questions ($n = .09$) has a much smaller contribution towards the percentage of the explained difference of the percentage of correct answers compared with the area of numbers ($n = .45$, $n = .37$) and measurements ($n = .34$). In all of the analyses the drop in achievements for the fourth quartile was, in general, smaller than the decline in the other quartiles. Perhaps one of the reasons for this is the "floor effect."

In conclusion, it appears that the Israeli mathematics curriculum for the lower secondary school must be brought into balance, and we must continue teaching quantitative subjects along with algebra and formal geometry. This change could help improve pupil achievements in quantitative subjects, and perhaps it could even enhance pupil achievements in algebra. The question we must ask is: in order to achieve such learning must we add hours to the curriculum? Interim results of an experiment we conducted indicate that one hour studying quantitative thinking instead of existing hours can drastically improve pupil achievements in quantitative subjects without jeopardizing pupil achievement in algebra. Furthermore, interim results show that adding an hour of quantitative thinking significantly improves pupil achievement in both quantitative subjects and in algebra.

References

Andersen, N. (1998). State School board oks back to basics for math. *Los Angeles Times*, p. A1.

Cockcroft, W.H. (1982). *Mathematics Counts.* London: Her Majesty's Stationery Office.

Department for Education and Skills (DfES). (2001). The Mathematics Strands of Key stage 3, 2001, *Framework for teaching Mathematics: years 7, 8 and 9.* DfES publications, United Kingdom.

Gal, I. (1999). *Numeracy, International Life Skills Survey - ILSS 2000*, Policy Reeserch Initiative. Statistics Canada

Gonzalez, E. J. & Miles. J.A. (2001). TIMSS 1999 User Guide for the International Database. Boston College.

Husen, T. (ed.), (1967). *International Study of Achievement in Mathematics:A Comparison of Twelve Countries.* Vol 1 & 2, Stockholm: Almquist & Wiksell.

Klein, D., & Milgram, R.J. (1999). L.A.'s math program just doesn't add up. *Los Angeles Times*, p. B7.

Manster, A.B. (2001). Mathematics and Numeracy: Mutual Reinforcement. In Lynn Arthur Steen (Ed.), *Mathematics and Democracy.* Woodrow Wilson National Fellowship Foundation.

Mullis, I., Martin, M., Gonzalez, E., Gregory, K., Garden, R., O'Connor, K., Chrostowski, S., & Smith, T. (2000). *TIMSS 1999 International Mathematics Report, International Study Center*, Boston College.

National Center for Education Statistics. (1993). *National Adult Literacy Survey - NALS.* United States Department of Education.

Organisation for Economic Co - Operation and Development (OECD). (2001). *Literacy Skills for the World of Tomorrow: Further Results from Pisa 2000*, Unesco Institute for Statistics.

Orrill, R. (2001). Mathematics, Numeracy and Democracy. In Lynn Arthur Steen (Ed.), *Mathematics and Democracy.* Woodrow Wilson National Fellowship Foundation

Postlethwaite, T. N. (1999), Overview of Issues in International Achievement Studies In B. Jaworski & D. Philips (Eds.), *Comparing Standards Internationally: research and practice in mathematics and beyond.* Oxford: Symposium Books.

Routitsky, A. & Zammit, A. (2002). *Association between intended and attained Algebra curriculum in TIMSS 1998/1999 for ten countries.* Australian Council for Educational Research.

Senk, S .L, & Thompson, D. R. (2003) *Standards Based School Mathematics Curricula.* Lawrence Erlbaum Associates, Mahwah, New Jersey.

Steen, L.A. (1997). *Why Numbers Count: Quantitative Literacy for Tomorrow's.* America. New York, NY: The College Board.

The Quantitative Literacy Design Team. (2001). The Case for Quantitative Literacy. In Lynn Arthur Steen (Ed.), *Mathematics and Democracy*. Woodrow Wilson National Fellowship Foundation.

The International Association for the Evaluation of Educational Achievement. (2000). Timss 1999 Mathematics Item, Percent of Responses by item Category for the *Mathematics Item Main Survey*. TIMSS International Study Center, Boston College.

Zuzovsky, R. (2001) *Israeli 8th Graders' Mathematics' Achievements Timss - 1999 Test Items.* Ramot Publication, Tel - Aviv University.

NOTES

1. TIMSS 1999 International Mathematics Report, ch-1 Exhibit 1.1.
2. TIMSS 1999 International Mathematics Report, ch-5 Exhibit 5.8.
3. Zuzovsky, R. Israeli 8 grades Mathematics Achievements TIMSS - 1999 Test Items pp. 60-63.
4. TIMSS 1999 International Mathematics Report, Reference-2 Exhibits R2.8 -R2.12.
5. Mathematics strand of the Key Stage 3 National Strategy, Department of Education and Employment. United Kingdom, 2001. Guide to the framework. p.10.
6. TIMSS 1999 International Mathematics Report, ch-6 Exhibit 6.15.
7. TIMSS 1999 International Mathematics Report, ch-3 Exhibit 3.1.
8. TIMSS 1999 International Mathematics Report, Appendix B Exhibit B.1.
9. TIMSS 1999 International Mathematics Report, Appendix B, Exhibit B.2.
10. TIMSS 1999 International Mathematics Report, Appendix B, Exhibit B.5.