Trends in Similarities and Differences of Students' Mathematics Profiles in Various Countries

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Abstract

The purpose of the article is to investigate how countries can be grouped according to their students' mathematics achievements. The patterns of students' answers for groups of countries are searched for, based on the difficulty of mathematics items for the students in those countries. The attempt is made not only to group countries according to their mathematics profile, but also to see whether the similarities within the particular group remain throughout the years. The main methods of the research include hierarchical cluster analysis and factor analysis. TIMSS (Trends in Mathematics and Science Study) 1995 and 2003 data is used for the analysis. Mathematics results of Grade 8 students of all participating countries are included. The investigation shows that although essentially the same groups remain throughout the three TIMSS cycles, but at the same time there are several interesting changes and the mathematics profile of all participating countries in the survey becomes more similar to each other.

Keywords: *TIMSS, trends, mathematics profiles, item difficulty, hierarchical cluster analysis.*

Introduction

International educational studies, including IEA Trends in Mathematics and Science Study (TIMSS), provide wide opportunities for comparisons of educational systems based on the students' achievements and background factors related to those achievements. We are interested not only to generally compare the achievements, but to see if there are groups of countries that can be seen as relatively similar or different. The richness of data collected by TIMSS allows searching for similarities based on a number of criteria. Schmidt (1997) attempted to group the participating countries by analyzing national curricula in mathematics and science. Japelj Pavešic and Korenjak-Cerne (2004) identified clusters of countries based on the teaching methods and attitudes towards mathematics reported by both teachers and students. Zabulionis (2001) searched for the clusters based on the relative difficulty of the mathematics and science items for the students in different countries. Later on, Grønmo, Kjærnsli, and Lie (2004) and Olsen (2005) continued in the same pattern as Zabulionis to look for cultural and geographical factors in patterns of responses to mathematics and science items.

From all studies we can see that there are certain groups or clusters of countries that can be identified through such analysis, and we can observe some clear geographical, language-based, and cultural groups. However, are these clusters constant throughout the time? TIMSS is a trend study that provides a unique possibility to look at how various patterns change, and not just the patterns in the general students' achievements. We know that since the first TIMSS cycle in 1995, many countries have undergone important political and economical changes, maybe even cultural changes in some regard. In a number of cases, drastic educational reforms have been implemented. Could this result in the shifts in the grouping of countries? Do the clusters of countries remain in essence the same throughout the years, or do they make completely different groups, or maybe some particular countries jump out from what seemed to be their natural group? Do countries (or clusters of countries) move closer to each other in this age of globalization and intense international cooperation and learning? And what could be the interpretation of the trends observed? These are the questions that we attempt to investigate in our study. Similarly to Zabulionis (2001), we are interested in the achieved curriculum rather than intended one as in Schmidt (1997) or reported teaching methods as in Japelj Pavešic and Korenjak-Cerne (2004).

Methodology and Data Sources

Countries certainly differ in their achievements on particular mathematics items. However, although average achievements might be different between some countries, their relative achievements on items can be similar. For example, a group of countries can perform relatively better on Algebra items than Statistics items, or opposite. These are the similarities that we are looking for, calling this relative performance a country's *mathematics profile*. The illustration of what the mathematics profile in this understanding means is provided in the figure No 1. In this case the "river" shows the item difficulties for Numbers' items in 1995. The items are put in the order of their average difficulty for all countries participating in TIMSS 1995 (the middle line represents this average difficulty). The upper and lower dotted lines show the highest and lowest item difficulties between the participating countries. The gray area illustrates the item difficulties between participating countries of one group that we call Post-Soviet countries. We can see very clearly that there is some pattern in relation to what items are easier or more difficult between these countries. Of course, some countries perform relatively better than the others. This can be taken care of by introducing relative difficulty of the items, standardizing them within countries by, for example, converting them to z-scores. However, for the sake of cluster analysis this is not needed, as clustering is based on correlation and is not affected by the generally higher or lower achievements, just by the pattern which we are especially interested in.

[include Figure 1 somewhere around here]

There is another aspect that also has been noted and addressed in a specific way in the works of Zabulionis (2001) and those who followed methodology developed by him. It is an issue of general item difficulty and its effect on the correlation. The fact is that items are generally easier or more difficult to all countries. Therefore, correlation based on general item difficulty will always be high, because all countries will perform better on easy items and worse on difficult items. This certainly produces undesired effect of very high correlation coefficients. To avoid this problem, Zabulionis (2001) and later Grønmo, Kjærnsli, Lie (2004) and Olsen (2005) standardize all items as well. In this way all items become equally difficult. Therefore, the "fact that some countries score higher than others and that some items are harder than others no longer appears in the data" (Grønmo, Kjærnsli, Lie (2004)). This certainly deals with the issue of unnaturally high correlation. However, we can see that it also produces another undesired effect, as by equalizing all items, a natural profile of the country can be lost, because it then very much depends on where in relation to average difficulty of the item specific country's item difficulty was. We illustrate this dilemma in the Figures 2 and 3 that are based on TIMSS 2003 data. We take two countries whose average achievements were rather different, and see their results (examples of profiles) on 15 random mathematics items. The Figure 2 is based on plain item difficulties, and Figure 3 is based on the z-scores of item difficulties (all items in this way are 'equally difficult'). We sort items according to their relative difficulties for Country 2. In the Figure 2 we can see that, although one country performs better than the other on all of the items, at the same time it is clear that relatively, countries perform very similarly to each other – the same items are more or less difficult for them. Their "profiles" are, therefore, similar. In the Figure 3, however, the patterns seem not to have anything in common, and the natural pattern / profile of the country's performance on various items is lost. Therefore, while choosing between the two problems, in this paper we decide to keep up the original mathematical profiles, and to cluster countries based on them, although we understand that in this case correlation coefficients are high because of the inherent item difficulty.

[include Figure 2 and Figure 3 somewhere around here]

In the article we provide graphical illustration of the clustering as well as correlation coefficients when doing factor analysis for the sake of comparing the coefficients and choosing which countries group within the clusters.

We use data from TIMSS 1995 and TIMSS 2003, Grade 8; mathematics. Having all the mathematics items' difficulties for all participating countries, we use a hierarchical cluster analysis with average linkage between groups. For this sake, we use Pearson correlation method. This is followed by the factor analysis in order to identify the factors that are specific to the groups. Here, we use Varimax method for factorization. We also investigate whether in general countries are drawing closer to each other. This is done by comparing differences between the correlation coefficients in 1995 and 2003.

Findings and Discussion

Results of clustering based on TIMSS 1995 mathematics profiles are displayed in Figure 4.

First clear group that emerges is comprised of English-speaking countries. England, Scotland, Australia, and New Zealand form one group which is closely linked to Canada, USA and Ireland group. A bit surprisingly, Netherlands jump in between. England with Scotland and Australia with New Zealand are closest pairs of countries based on this clustering.

Another clear group consists of Western Europe countries: Norway, Sweden, Iceland, Austria, Germany, Denmark, and Switzerland. Together with the English speaking countries they form one bigger group that represent Western civilization.

Post-Soviet countries represent another big group of countries that cluster together. It includes Latvia, Lithuania, Russia, Czech Republic, Slovakia, Slovenia, Hungary, Romania, and Bulgaria.

The Post-Soviet group of countries is closest to another, although smaller, group of Spanish-Portugal culture countries: Spain, Portugal, and Colombia.

Greece, Israel, and Cyprus also form a little group of countries that cluster together. A similarly small group is formed of two Belgium communities (French and Flemish) and France. They are relatively close to each other and then cluster with Post-Soviet and Spanish-Portugal groups.

All the groups mentioned above then form a big one cluster. The other two small clusters are more different from them. One of these clusters consists of well-performing Far-East countries: Japan, South Korea, Hong Kong, and Singapore. The other small cluster consists of a bit diverse, but having some similar characteristics group of countries: Iran, Thailand, South Africa, Philippines, and Kuwait.

By analyzing factors within the big groups of countries, the following interesting results were obtained. Table 1 shows that the group of Western Europe countries naturally divide into two sub-groups of Nordic and German speaking countries.

[include Table 1]

Table 2 divides the Post-Soviet group of countries. We can see two rather distinct sub-groups with Romania and Bulgaria being a bit different from both of them. The first of the two sub-groups include Latvia, Lithuania and Russia – countries that in 1995 were still emerging from the same educational system, and the second one consists of Czech, Slovakia, Slovenia, and Hungary – again very close group of countries, although a bit distinct from the previous one.

[include Table 2]

We are especially interested to see whether general grouping of countries as well as factors within the groups change throughout the time. For this we use the database of TIMSS 2003, the latest available information, and repeat similar analysis as with TIMSS 1995. We meet a challenge of the change in the participating countries. However, we can still see the main patterns in grouping.

In Figure 5 we see that English-speaking countries remain together in a very close cluster. The Western Europe group of countries, although smaller than in 1995, also holds together. Post-Soviet group of countries, however, became larger than in 1995, but, despite the various reforms that were carried out in them, still makes one big cluster. The Far-East well-performing countries, joined by Chinese Taipei, again remained together. Cyprus and Israel, although without Greece this time, still cluster together in a small separate group.

However, a new group of Islamic countries emerged consisting of Bahrain, Jordan, Iran, Egypt, Morocco, Tunisia, and Lebanon.

A group of developing countries this time divided into two clusters, one of which, including Botswana, Philippines, Indonesia, and Malaysia, is closer to Post-Soviet and Islamic countries, and the other one, consisting of Ghana, South Africa, and Saudi Arabia, is rather distinct from all other countries.

It is interesting that Armenia that joined TIMSS in 2003 seems to have a distinct mathematics profile from all other countries as it does not group with any of them and is far from all of them.

We were also interested to see if the factors within the big groups also remained the same after the 8 years. However, the Western Europe cluster became rather small, therefore, we are only investigating the Post-Soviet block of countries. This is the cluster that we are especially interested in. This group during this time expanded in TIMSS, but it is also a group of countries that was undergoing many changes, both in their political, economical, and social life, but also in their educational systems, in many cases implementing large educational reforms, changing curricula, textbooks, teacher training. If before most of them had very similar or even the same educational systems, after the dissolution of the Soviet Union, they were attempting to develop their own, modern education systems. Despite this fact, as mentioned above, we see that all these countries still make one large group. Within this group, however, in 2003 factors work a little bit differently than in 1995. In Table 3, we see that 3 distinct sub-groups are formed in Post-Soviet countries group. All of the three groups are very much based on geographical clusters. As in 1995, neighbouring countries of Slovenia, Slovakia and Hungary stay together in one sub-group, although this time without Czech Republic that did not participate in TIMSS 2003. Russia together with the group of geographically and culturally close countries (Serbia, Romania, Moldova, Bulgaria, and Macedonia) makes another of the sub-groups. It is interesting that Latvia and Lithuania this time group not with Russia as in 1995, but with Estonia, forming a new Baltic countries sub-group. It is worthy to note that Estonia's general performance in TIMSS 2003 mathematics is considerable higher than those of Latvia and Lithuania, but, despite this, the general mathematics profile of these three countries is close.

Although we see that similar clusters of countries continue to form, we also notice that there is a general tendency of countries drawing closer to each other in their mathematics profile. For this analysis we take only the countries that participated in both TIMSS 1995 and 2003, and calculate the differences between the correlation coefficients based on their mathematics profiles. The results of these calculations are provided in Table 4. The positive differences between the 2003 and 1995 correlations are marked with grey pattern. Although we do not calculate statistical significance of the differences, but the general tendency is clear – the table seems rather grey. Can this be attributed to the tendencies of globalisation, countries coming closer to each other in their educational culture, including teaching and learning mathematics? Or is this the influence of TIMSS for participating countries? We are aware of the cases when the country's educational reform in mathematics was strongly influenced by TIMSS philosophy and framework. It is not illogical to assume that in other countries that participated in TIMSS this survey in one way or another would have had an impact on their educational systems.

Conclusions

The analysis showed that:

1. The mathematical profiles based on geographical and cultural patterns seem to remain the same as countries form similar clusters in both 1995 and 2003.

2. However, there is also a general tendency of countries drawing closer to each other in their mathematical profiles which can be a result of globalization or "TIMSSation".

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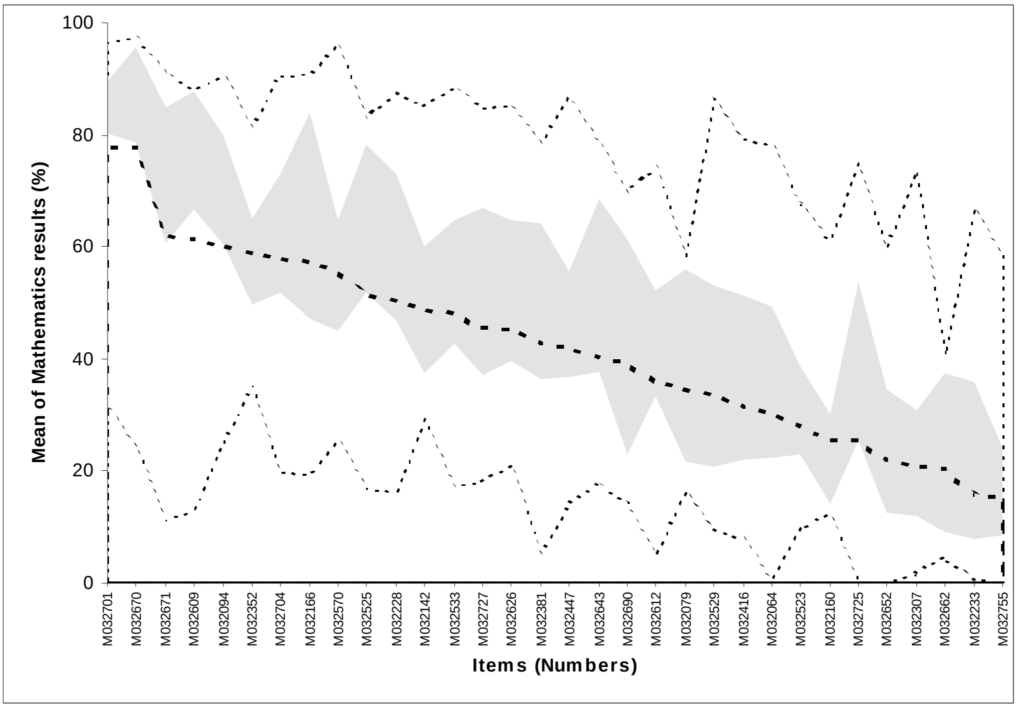

Fig. 1: Example of mathematics profile of a group of countries

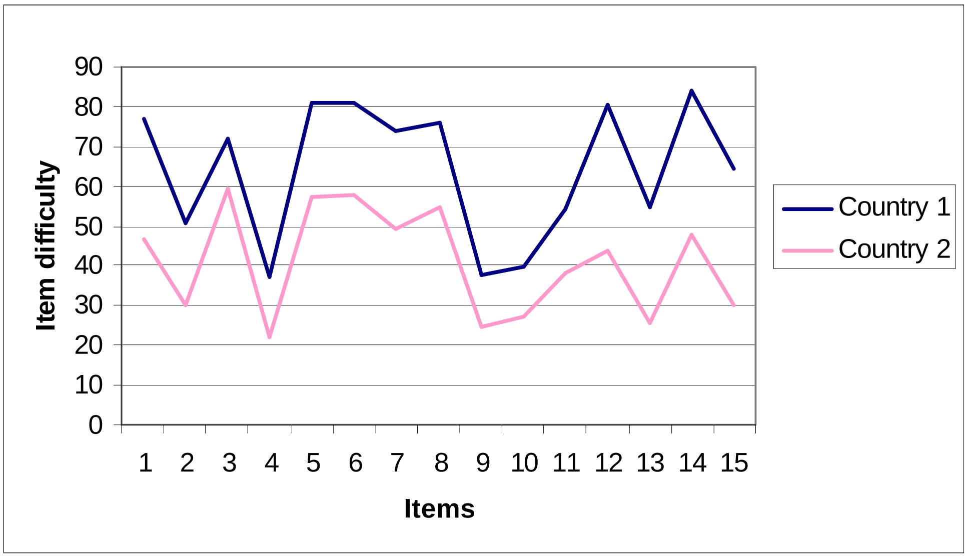

Fig. 2: Mathematics profiles of two countries based on item difficulty of 15 items

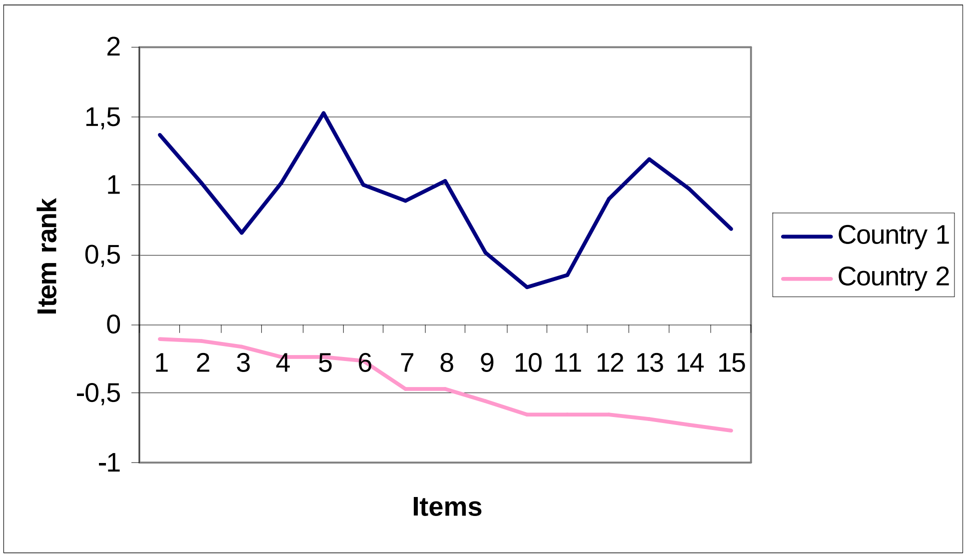

Fig. 3: Mathematics profiles of two countries based on item rank of 15 items

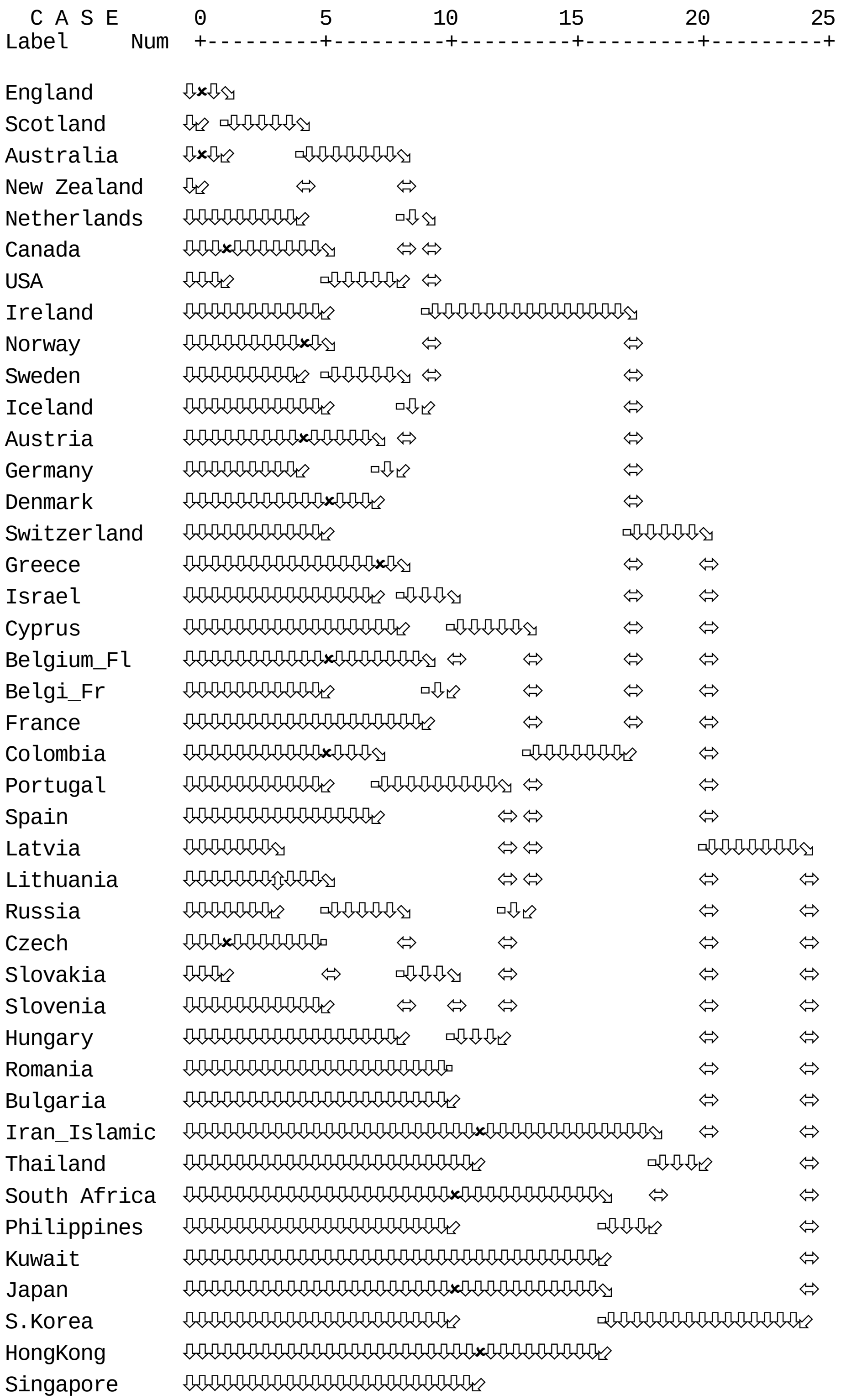

Fig. 4: TIMSS 1995 cluster dendogram based on mathematics profiles of participating countries

Table 1: TIMSS 1995 Western Europe cluster's factorization into two sub-groups

	Component	
	1 (Nordic)	**2 (German)**
Norway	**0,759**	0,597
Sweden	**0,869**	0,432
Iceland	**0,856**	0,451
Denmark	**0,702**	0,637
Austria	0,403	**0,895**
Germany	0,561	**0,800**
Switzerland	0,643	**0,713**

Table 2: TIMSS 1995 Post-Soviet cluster's factorization into four sub-groups

	Component			
	1	**2**	**3**	**4**
Latvia	0,463	**0,651**	0,401	0,378
Lithuania	0,470	**0,711**	0,318	0,372
Russia	0,481	**0,613**	0,479	0,320
Czech	**0,707**	0,550	0,283	0,255
Slovakia	**0,694**	0,483	0,408	0,266
Slovenia	**0,651**	0,426	0,456	0,325
Hungary	**0,792**	0,312	0,290	0,379
Romania	0,360	0,344	**0,784**	0,353
Bulgaria	0,373	0,357	0,368	**0,768**

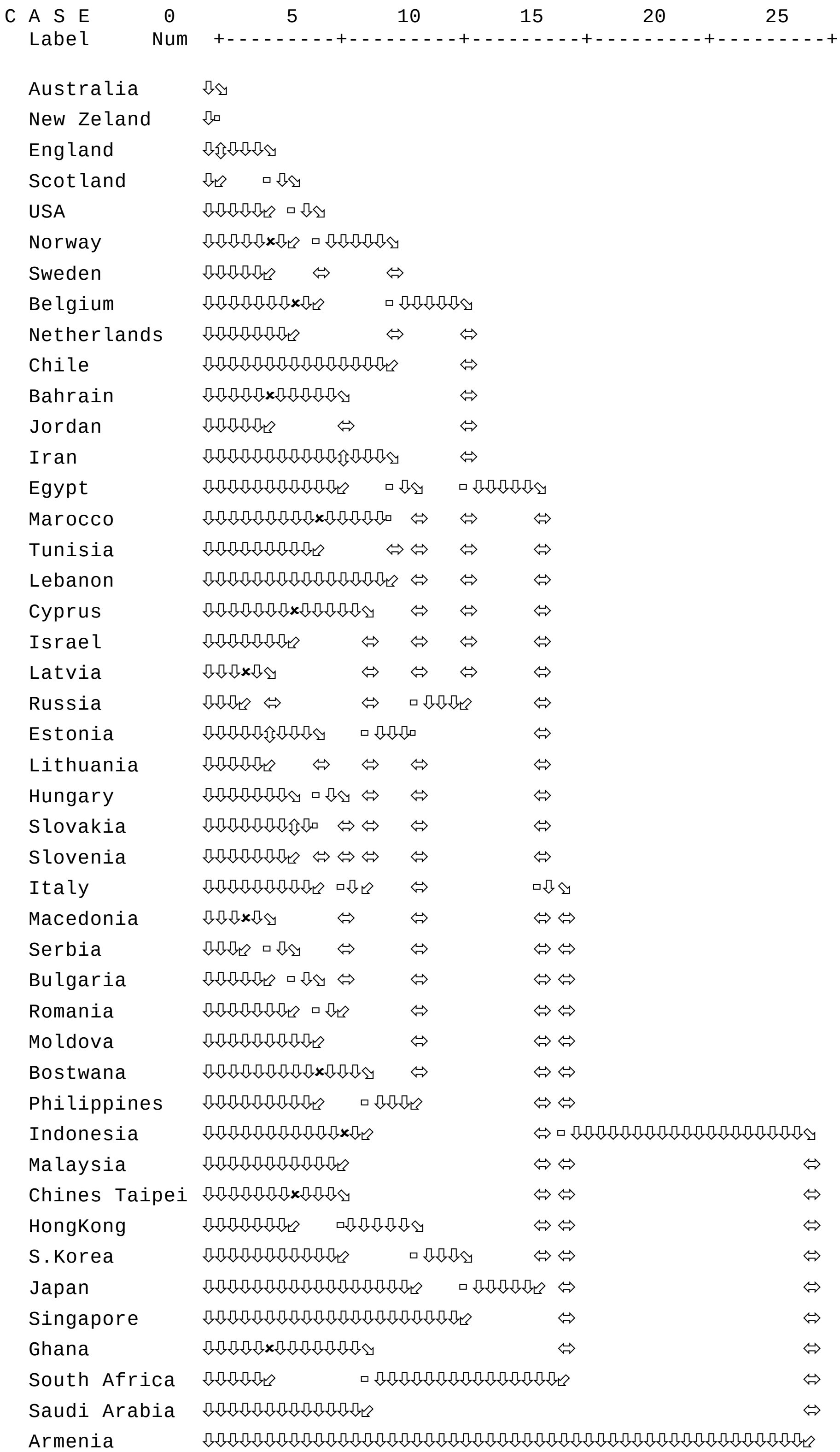

Fig. 5: TIMSS 2003 cluster dendogram based on mathematics profiles of participating countries

Table 3: TIMSS 2003 Post-Soviet cluster's factorization into three sub-groups

	Component		
	1	2	3
Latvia	0,503	**0,641**	0,518
Lithuania	0,481	**0,716**	0,419
Estonia	0,449	**0,709**	0,465
Hungary	0,378	0,423	**0,797**
Slovakia	0,561	0,486	**0,602**
Slovenia	0,456	0,578	**0,587**
Russia	**0,635**	0,559	0,442
Macedonia	**0,758**	0,429	0,387
Serbia	**0,760**	0,343	0,489
Bulgaria	**0,688**	0,561	0,370
Romania	**0,795**	0,319	0,428
Moldova	**0,800**	0,451	0,234

Table 4: Differences between the correlation coefficients based on mathematical profiles of countries participating in TIMSS 1995 and TIMSS 2003

	Australia	Belgium	Bulgaria	Cyprus	England	HongKong	Hungary	Iran	Israel	Japan	S.Korea	Latvia	Lithuania	Netherland	New Zelan	Norway	Philippines
Australia	*																
Belgium	0,006	*															
Bulgaria	0,004	0,038	*														
Cyprus	0,048	-0,018	0,085	*													
England	0,028	0,088	0,029	0,125	*												
HongKong	-0,021	-0,006	-0,002	-0,023	0,095	*											
Hungary	0,035	0,038	0,009	-0,007	0,103	0,000	*										
Iran	0,040	0,038	0,109	0,044	0,111	0,026	0,040	*									
Israel	0,002	-0,001	0,086	0,006	0,073	0,015	0,008	0,077	*								
Japan	-0,008	0,048	0,059	-0,062	0,085	-0,087	0,007	0,023	0,019	*							
S.Korea	-0,060	0,022	0,045	0,018	0,023	0,040	0,037	0,057	0,064	-0,009	*						
Latvia	0,057	0,065	0,068	0,088	0,089	0,028	0,028	0,070	0,063	0,018	0,080	*					
Lithuania	0,102	0,065	0,040	0,128	0,155	0,020	-0,011	0,107	0,067	-0,001	0,026	-0,019	*				
Netherlands	-0,020	0,043	0,000	0,052	-0,015	0,034	0,060	0,071	0,033	0,029	-0,052	0,053	0,140	*			
New Zeland	0,006	0,023	0,039	0,057	0,002	0,035	0,065	0,062	0,011	0,064	-0,007	0,067	0,130	-0,043	*		
Norway	-0,005	-0,030	0,014	0,034	0,038	-0,056	-0,004	0,109	-0,016	-0,003	-0,059	-0,036	0,001	0,003	0,010	*	
Philippines	-0,030	-0,022	0,155	0,074	-0,016	-0,021	0,010	0,024	0,053	-0,020	0,043	0,056	0,059	-0,069	-0,045	-0,082	*
Romania	0,018	-0,030	0,072	-0,027	0,088	-0,012	0,002	0,028	-0,006	-0,094	0,015	0,004	0,002	0,050	0,063	0,001	0,136
Russia	0,016	0,027	0,073	0,052	0,083	0,007	-0,003	0,071	0,020	-0,034	0,042	-0,002	-0,027	0,072	0,055	-0,035	0,084
Scotland	0,015	0,068	0,048	0,087	-0,012	0,058	0,083	0,102	0,043	0,077	0,035	0,089	0,147	-0,025	-0,020	0,019	-0,045
Singapure	0,051	0,091	0,142	0,065	0,177	0,045	0,092	0,084	0,096	-0,041	0,023	0,161	0,159	0,137	0,090	0,045	0,105
Slovakia	0,035	0,037	0,073	0,042	0,086	-0,023	-0,021	0,110	0,047	-0,022	0,065	0,014	-0,032	0,071	0,067	0,002	0,095
Slovenia	0,068	0,023	0,033	0,037	0,139	-0,073	-0,012	0,066	0,017	0,001	0,012	-0,006	0,002	0,092	0,104	0,045	0,077
SAR	-0,084	-0,066	0,049	0,052	-0,056	-0,070	-0,085	0,073	-0,015	-0,063	-0,037	-0,068	-0,014	-0,073	-0,079	-0,034	-0,007
Sweden	0,030	0,019	0,061	0,090	0,057	0,039	0,086	0,119	0,053	0,086	0,003	0,048	0,087	0,012	0,015	-0,008	-0,089
USA	0,023	0,031	0,081	0,047	0,061	0,000	0,059	0,013	0,022	-0,031	-0,036	0,096	0,101	0,033	0,019	-0,045	-0,064