**Paper for; The 3rd IEA International Research Conference, IRC-2008. Science Education Center of National Taiwan Normal University, September 18-20, 2008.**

# Is multilingualism to count on?
# The importance of instructional modes for multilingual students' mathematical progress.

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## Abstract

*In this study the importance of instructional modes for multilingual students' mathematical progress is discussed and how to conceptualise and define "instructional modes" is analyzed. Findings from a study of the educational situation in Sweden are presented. A measurement model for instructional modes in mathematics education is being explored. The traditional division in teacher- versus student-centred modes of instruction is challenged by an alternative perspective focusing on teachers' intervention and students' interaction. Results from this study support the possibility to adopt this alternative perspective on modes of instruction.*

## Study Purpose

The aim of this study is to discuss the theoretical starting-points in categorisation of instructional modes relevant for the mathematical education of multilingual Swedish students in 8th grade, and to explore a measurement model for analyzing these modes. In a further study it will be explored how these different instructional modes interact with different group compositions, in order to identify in what way multilingual students may gain in mathematical achievement. It will then be explored in what way different instructional modes could be supportive for students' mathematical progress. Findings from TIMSS 1995, 2003 and 2007 will then be used. In this study data from TIMSS 2003 have been analyzed.

## Background

### Why is this study important?

As indicated by TIMSS there has been a dramatic decline in mathematics achievement from 1995 to 2003 in Sweden, which suggests that there is a great need for developing mathematics education in the Swedish compulsory school. Besides the generally unsatisfactory results there is also a great variation between classes, schools and communities. It has been established that the socioeconomic, linguistic and ethnical segregation in the schools has continued to increase in the 21st century (Gustafsson, 2006) and the share of students without certificates in mathematics is large among multilingual students. The previous policy to ensure education for all students has been replaced by a policy with compensatory approaches, but still there is great variation between schools and groups. More knowledge is needed concerning successful instructional modes for different groups of students.

Students with different ethnical backgrounds are less successful in mathematics in the compulsory school than their Swedish classmates. Among 9th graders, the average mark of those with different ethnical backgrounds was in 2005 almost 20% lower compared to the Swedish students. The problem is most evident in metropolitan areas where the number of students with no final mark in mathematics often is twice as large as compared to the average of the country as a whole.

### Multilingualism

What is meant by multilingualism? Grouws (1992 ) gives two different definitions. One describes students with another language than Swedish in their social environment, where the second language is presupposed to have influenced the Swedish language and academic performance of the student negatively. This definition agrees well with that presented by the Swedish school authorities.

The second definition emphasizes that language is contextually constructed and that the students are supposed to have competences within one context but not within others. ”Socially crated groups exist somewhere between the individual and the larger society. It should follow that the mathematics achievement of such groups should be interpreted as a social issue rather than as a matter of individual differences” (Grouws, 1992 p. 627). This study is built on the latter definition. When analyzing how structural phenomena, instructional modes and group compositions, covary with mathematical achievements, these achievements of the multilingual students are primarily viewed as dependent on the context and not on their multilingualism.

**Instructional modes**

The present study highlights problems with different achievements by examining the importance for multilingual students to receive different instructional modes in mathematics education in the compulsory school, grade 8. Instead of regarding the differences between different groups of students as dependent on their backgrounds, they are in this study viewed as dependent on e.g. the performance of the mathematical education; ”…differences in mathematical achievements among groups do not rest solely upon students´ cultural/mathematical backgrounds, but also in the socio-political organization of mathematics classrooms” (Lester, 2007, p. 407).

A widespread way to characterize mathematical education is to distinguish between teacher- and student-centred modes of instruction. The ”traditional instructional mode” is characterized by the teacher-centred instruction where the teacher mainly explains procedures and gives directions (Hiebert et al., 2003; Porter, 1989; Silver & Smith, 1996; Stigler & Hiebert, 1997). The students are supposed to listen and remember what the teacher says. Very little time is spent on letting the students explain their thinking, show their work and to reaching consensus about mathematical ideas. Procedures are dominating and chosen before conceptual learning. When this instructional mode is related to theories about the importance of interaction and teacher-intervention for students’ opportunities of learning mathematics, one can ascertain that interaction is not prominent.

The student-centred instruction is thought to be characterized more by interaction in learning, which is meant to develop the mathematical identity of the student (Ball & Bass, 2000; Boaler & Greeno, 2000; Cobb, Wood, & Yackel, 1993; Lampert, 2001; Yackel, Cobb, Wood, Wheatley, & Merckel, 1990). The teacher is viewed as very important for initiating this interaction and to strive for high quality in the conversation (Yackel et al., 1990). So, the teacher is supposed to be active and creative and to intervene in different situations where the dialogue is central. A question from the teacher is meant to structuralize the dialogue in such a way that all students get the opportunity to cooperate in a mathematically meaningful way. Research shows that students are learning better if conversation is a feature in the learning process (A. L. Brown & Palinsar, 1989; King, 1992; Peterson, Janicki, & Swing, 1981; Saxe, Gearhart, Note, & Paduano, 1993; Webb, 1991; Yackel et al., 1990). It has also been shown that when students are listening to other students, they learn better if the teacher is well prepared and leads the conversation (Kiran, 2002). All together it has been revealed that student-centred instructions support students’ mathematical learning when there is an interaction and when the teacher is active. For multilingual students this is a prominent condition.

Previous ideas, suggesting that traditional teacher-centred modes of instruction are more appropriate for low performing students than more student-centred modes, are challenged in some international research reports where the latter appears to support all students in a more successful way (Zuzovsky, 2008). Zuzovsky writes; “The narrowing school achievement gap between the Jewish and Arab populations in Israel is, in the first place, a result of teachers´ effects on their classes due to adopting and adapting suitable modes of instruction at the class level” (p. 67). These suitable modes Zuzovsky talks about are described in the following way: “In the earlier study, I concluded that traditional teacher-centred modes of instruction were more beneficial to Arabic-speaking students. This time, the type of instruction that favoured students in Arabic-speaking schools, also included student-centred and inquiry-oriented instructional activities”. (p. 66).

When analyzing the potential of different instructional modes in the Swedish math education, the dimensions student- and teacher-centred could however be misleading. Critical dimensions, like interaction among peers and teacher-intervention, are not always apparent. Inasmuch Sweden often is regarded as a country with student-centred instruction, the existence of interaction and teacher-intervention ought to be analysed. This study is a contribution to this analysis.

**Differentiation**

In Sweden, comprehensive school is characterized by a large amount of student-centred modes (Vinterek, 2006), due to a large frequency of students' "independent work" where the teacher intervention is low. This dominance has become apparent since 1994 when tracking was excluded from the curricula[1]. Despite the fact that mathematics education formally is untracked in Sweden, the pattern of results with a low average and increasing dispersion, is similar to countries with an early differentiation (Hanushek & Wössmann, 2006). For this reason it should be of interest to further examine student-centred modes of instruction in the Swedish mathematics education and especially for multilingual students. The focus of this study is on the teacher intervention of the instructional modes.

**Challenging the traditional division**

The theoretical starting point for the categorization of the instructional modes, student- and teacher-centred, which Zuzovskys´ study (2008) was based on, is problematized by other researchers. In East Asian regions it has been shown that descriptions of mathematics education classrooms cannot easily be done in terms of teacher- and student-centred modes: "… these descriptions fail to capture many salient features of pedagogy" (Mok & Morris, 2001). Despite teacher-centred instructions, the authors write, the teachers succeeded with a "social constructivist/Vygotskian model", where the instructions were impressed by interaction and the active participation of the pupils. Classroom contexts were characterized by a combination of whole class teacher-pupil interactions and highly structured group/pair work.

A later study by Mok (2003) showed that the teacher-centred instruction was characterized by a conscious teacher intervention together with students' active thinking moments. These two studies thus showed that there were important features of interaction and teacher intervention in teaching considered to be teacher-centred. Also Zuzovskys´ (2008) study challenges traditional beliefs about which of the teaching models support different pupils' learning in mathematics. This confirms the need of finding the underlying dimensions/aspects of traditional instructional modes.

The purpose of a conceptual categorization of mathematics education is to make possible an understanding of how to determine whether or not the essential conditions for pupils' learning of the subject are achieved. For multilingual pupils' learning the communicational processes are of significant importance.

It has been shown that teachers make use of the variation between students and take charge of peer effects and enable interaction (J. Moschkovich, 1999; J  Moschkovich, 2002; Shayer & Adhami, 2007). Moschkovich writes that all ways of talking "can contribute in its own way to the mathematical discussion and bring resources to the conversation" (1999, p. 12).  Of course, it is of importance what skills multilingual students have in their languages and how they make use of their languages. This is illuminated in a theory of threshold effects, (Cummins & Swain, 1986), and in theories about code switching, (Setati & Adler, 2000).

Theories dealing with the composition of the group are of interest, such as the advantages achieved with a wide span of performance capacity within the group, specially when teachers make use of this span in the teaching (Barwell & Clarkson, 2004; Hanushek & Wössmann, 2006; Shayer & Adhami, 2007). The group composition also is pointed out as essential for the collaborative learning (Dillenbourg, 1999), which is based on the importance of the interaction within the teaching groups.

The teacher-intervention is here regarded as an important presumption and it includes different grades and qualities of the collaborative elements in the education.

Theoretical starting points for conceptual categorisations of instructional modes are interaction and group compositions. These constitute basic requirements when language skills and experiences are

[1] Tracking was officially excluded 1980, but in practice it was continuing until 1994.

taken into consideration in mathematic learning. The acting of the teacher, teacher-intervention, is of importance for how those variables interact (Bentley, 2003; Stigler & Hiebert, 1999).

### This study

In this paper a measurement model of instructional modes in Swedish mathematics education will be presented in an attempt to answer the question as to which instructional dimensions are apparent. This study is based on TIMSS 2003 data and it will identify indicators that delineate these dimensions. The results are discussed both towards the traditional differentiation, student- or teacher-centred instruction, and towards supportive theories for multilingual students' mathematical learning. Results from this study will form the basis of the later structural equation modelling analysis (SEM).

## Method

The study is realized as a secondary analysis of TIMSS data from 2003 focusing on mathematics for Swedish students in 8th grade. A latent variable analysis was conducted in order to identify descriptive dimensions of instructional modes which may support multilingual students' mathematical progress. It is hypothesized that modes sustaining language as an active element of the learning process and modes that make use of the full range of performance and language capacity within a group are characteristic of such approaches.

### Latent variable analysis through exploratory factor analysis, EFA

In the first step the aim was to discover and define latent variables and a measurement model that could provide the basis for the further confirmatory analysis of relations among the latent variables. "The strategy was to solve for a series of uncorrelated general factors of decreasing size, each accounting for as much as possible of the covariation left unaccounted for by the preceding factors" (Loehlin, 2004, p. 169). EFA, as a data driven approach, aims to describe the proper number of common factors and to uncover which ones of the observed variables that are reasonable indicators of the latent dimensions (Brown, 2006). A multilevel exploratory factor analysis, MEFA, was realised for the analysis of the student-data and traditional EFA for the teacher-data. The statistical program packages used were Mplus (L. K. Muthén & Muthén, 1998) and STREAMS (Gustafsson & Stahl, 2005), where the latter acted as a pre-processor.

#### *Analysis of the student-data*

A first step in developing a measurement model involved performing an exploratory factor analysis on data from the student questionnaire. This analysis investigated both the underlying structure of the items by defining a set of common latent dimensions, or factors, and the associations between the items yielding these latent variables. In this study dimensions of instructional modes are represented in latent variables. Another procedure used is to aggregate factor scores to the group level, which here is the classroom level. This approach has frequently been applied in the educational effectiveness literature (D´Haenens, Van Damme, & Onghena, 2008). But the authors say there are several shortcomings related to this method: dependence between observations often leads to overestimated interitem correlations or covariance, biased factor scores, and the assumed absolute correspondence between the individual underlying dimensions and the group-variables often does not exist. Instead of the standard factor analysis plus aggregation, D´Haenens et al. recommend using multilevel explorative factor analysis, MEFA, as this method separates the collective part of the perceptions from the individual part. This is a similar method to multilevel confirmatory factor analysis, MCFA (B. O. Muthén, 1994), proposed by Van de Vijver & Poortinga (2002). D´Haenens et al. (2008) ascertain that "it should be evident that an MEFA is strongly advised when confronted with a dependency present in your dataset" (p. 22). For this study the analysis of student-data MEFA was applied and conducted in Mplus and STREAMS.

#### *Analysis of the teacher-data*

A next step in developing a measurement model was to analyse the teacher data. Dimensions of instructional modes are indicated by observed variables representing dimensions of instructional modes from the teachers´ point of view. For the purpose of improving the analysis a next step involved data-reduction through item-parcelling. Homogenous packages were constructed.

**Latent variable analysis through confirmatory factor analysis, CFA**

In the next step confirmatory factor analysis (CFA) was applied. In CFA both the number of factors and the pattern of indicators have to be specified in advance, which requires the researcher to be well acquainted with the research field, "Thus, unlike EFA, CFA requires a strong empirical or conceptual foundation to guide the specification and evaluation of the factor model" (Brown, 2006, p.14). Because of the design effect in survey research it was necessary to account for the effect of the social context on the individual responses, that is the hierarchical structure of the population (Hox, 2002). Multilevel Confirmatory Factor Analysis was therefore conducted.

***Analysis of the student-data***

This first step of the two-level CFA both used results from the previous EFA-study and theoretical foundations to specify a factor model for how to uncover the structure of factors and observed variables of the student-data. For this purpose, a two-level structural equation model approach was adopted and the Mplus (L. K. Muthén & Muthén, 1998) and STREAMS (Gustafsson & Stahl, 2005) software applied in the analyses.

The model specified above was a two-level model for students nested in classes and to measure the instructional modes twelve items from the TIMSS 2003 student questionnaire were used, see Table 1. Students in the TIMSS study responded to the questions "In math lessons, how often do you do…?" Responses were indicated on a 4-point Likert scale, which ranged from 1 (Every or almost every lesson) to 4 (never).

***Analysis of both the student- and teacher-data***

Instructional modes were in this second step of the analysis measured by including items from the teacher questionnaire, see Table 1. Teachers in the TIMSS study responded to the questions "In math lessons, how often do you ask students to..."? Responses were noted on a 4-point Likert scale, which ranged from 1 (Every or almost every lesson) to 4 (never).

Several indices were used to assess model fit; *chi-square test*, *root-mean-square error of approximation* (RMSEA) and the *standardized root mean-square residual* (SRMR) (Brown, 2006). RMSEA values less then 0.05 represent a "close fit", and models with values above 0.1 should be rejected. The SRMR was used as an absolute fit index. The SRMR value should be 0.08 or less. Because the chi-square statistic is very sensitive to sample size, chi-square/df ratio was examined to check fit (Kline, 1998). A goodness-of-fit index, *Comparative Fit Index*, CFI, shows a good performance overall (Hox, 2002). Usually a value of at least 0.95 is required to accept a model.

[Take in Table 1 about here]

## Data Sources

The data source for this study was obtained from the TIMSS 2003 studies in Sweden.
4 256 students participated in the study and they were from 274 classes in 160 schools.

The contextual variables were derived from teacher and student questionnaires. These variables are viewed as process-variables and describe both students´ and teachers´ opinions of the implementation of the mathematics education, of instructional modes and also their self-assessment in a range of related questions.

In the data subset used, only those classes with one mathematics teacher were included, which were 253 out of 274. After listwise deletion among those classes, there were 3 288 observations left, in 217 classes with an average cluster size of 15.15. Observed variables in the MEFA on student-data, are a collection of variables concerning which and how much mathematics students meet up during the mathematics education, in which way the teacher instructs and also interactions between peers. In Table 2 the 14 variables used are shown.

[Take in Table 2 about here]

Observed variables used in the EFA on teacher-data constitutes a range of variables concerning teachers opinions about teaching techniques they use when conducting the instruction (e.g. teacher activity, interaction), their self-assessment of competence as mathematics teachers and variables concerning social behaviour in the classroom. In Table 3 the 65 variables used are shown.

[Take in Table 3 about here]

## Findings and Discussion

### Exploratory factor analysis, EFA

Below follow the findings for the student-data and the teacher-data separately.

#### *Student-data*

In order to investigate how variables observed on the student level define a set of dimensions of instructional modes, a multilevel explorative factor analysis (MEFA), was conducted based on students grouped within classes. The analysis produced separate factor solutions for student- and class-levels. The test of model fit gave an RMSEA of 0.025.

#### *Individual-level factor solution for student-data*

Table 4 shows the inter-factor correlations for the three-factor solution obtained for the pooled-within data and Table 5 shows the pattern coefficient matrix.

[Take in Table 4 about here]

[Take in Table 5 about here]

The first factor includes items indicating students´ experiences of content topics in mathematics education: “how much mathematics is being taught within the mathematics education and which fields are the most common”? This factor was labelled *Mathematics content.* The second factor includes items dealing with instructional characteristics. In this dimension it became apparent not only if the teacher teaches and explains, but also if the teacher organises and arranges the working models and how these models are formed. Factor two was thus interpreted as a dimension of *Teacher intervention*. The third factor is dominated by items reflecting students´ opinions about the interaction with other peers in the instructional group. The dimension is also indicated by items related to how the content is explained to the student. This is of course related to the structure of interaction between peers. The factor was labelled *Peer-interaction*.

#### *Group-level factor solution for student-data*

Table 6 shows the factor correlations for the three-factor solution obtained for the between-group analysis, and Table 7 shows the pattern coefficient matrix.

[Take in Table 6 about here]

[Take in Table 7 about here]

In the class-level analysis a three-factor model was obtained as well. The first factor is similar to factor one in the Within-analysis, *Mathematics content*. It includes items indicating students´ experiences of content topics in mathematics education. The item BSBMHASM ("How often do you practice addition, subtraction, multiplication and division"?) generated a so-called Heywood case with a factor-loading larger than unity, which may indicate an empirically underidentified model (T. A. Brown, 2006). The second factor in the between solution as well as in the within solution, contains items about instructional characteristics, *Teacher intervention.* What differentiates the between-analysis from the within-analysis is not the character of items but rather the number of items. There is a broader range of items indicating instructional characters. The third and last factor is also in the Between-analysis dominated by the interaction between peers in the education group, *Peer-interaction*. As for the second factor, there are more, rather then other variables, indicating the dimension in the between-analysis compared to the within-analysis.

***Improving the EFA solution for the student-data***

To examine the possibility of simplifying the two-level EFA results, an attempt was made to aggregate factor scores and also to reduce the number of observed variables through parcelling observed variables. However, aggregation of factor scores was not feasible, because Mplus does not offer the option of computation of factor scores under MEFA. An attempt at data-reduction through item-parcelling was conducted instead, but without any success to enhance model fit, see Table 8. One reason for this was that the two-level model increased the complexity of item parcelling.

[Take in Table 8 about here]

***Discussion***

Although EFA is not hypothesis-driven, it is an important tool as it helps the researcher to get acquainted with the empirical field of research, and to get a basis for judging if the indicators are theoretically interesting or not. Furthermore, with MEFA differences between levels are accounted for, so the variations noticed in the observed variables, can with greater assurance be connected to the theoretical constructs.

It is not surprising that the dimensions of instructional modes identified in the Swedish mathematics education are *Mathematics content*, *Teacher intervention* and *Peer-interaction*. It has been argued that too much time is spent on other aspects than on the mathematics content in mathematics education, so this aspect of instructional modes was expected to vary between classes and schools. The recently popular student-centred instructional mode which in Sweden is called "independent work", has for a long time been both acclaimed and criticised. The dimension *Teacher intervention* is likely to be a reflection of this. The third dimension highlights in what way the mathematics education is influenced by students working in their own pace, or by making use of the "peer-effect" in collaborative working, which also has been in focus when the Swedish mathematics education is described and discussed.

***Teacher- data***

In order to investigate how items in the teacher questionnaire define dimensions of instructional modes, traditional exploratory factor analysis, EFA, was conducted both as a "single-item approach" and as an "item-parcelling approach". With the aims of simplifying the model and improving model fit, data-reduction was accomplished by making parcels of items into homogenous packages. The primary potential advantage of using parcels in this analysis was that this method offers improves reliability and relationships with other variables. Models based on parcels may be considerably less complex than models based on individual items (T. A. Brown, 2006, p. 408). One problem using parcels could arise if the underlying structure of the items in a parcel is not unidimensional. This again demonstrates the importance of the researcher being familiar with the empirical field.

***The "single-item-approach" for teacher-data***

The item-level analysis produced a model with four common factors. The test of model fit gave an RMSEA of 0.084. Table 9 shows the factor correlations for the factor solution proposed, and Table 10 shows the factor loadings. This model included two theoretically significant dimensions, both due to how teachers perceive their own competence. No dimensions reflecting instructional modes were salient. One possible explanation for this is that it may be difficult for teachers to characterise their own ways of approaching instruction. It also is possible that more information can be obtained from the answers if data is reduced to a less complex structure.

[Take in Table 9 about here]

[Take in Table 10 about here]

***The "item-parcelling-approach" for the teacher data***

Data-reduction through item-parcelling was carried out to achieve a less complex structure. Table 11 shows items used in the analysis. Parcels have to correspond both with the structure of factor-loadings, se Table 10, and with the meaning of the theoretical constructs.

[Take in Table 11 about here]

Table 12 shows the factor correlations and Table 13 shows the factor loadings for the "item-parcelling-approach". The test of model fit gave an RMSEA of 0.036.

[Take in Table 12 about here]

[Take in Table 13 about here]

In this three-factor model there are dimensions with different characteristics. The first factor includes items indicating the very most basic competences assigned to mathematics teachers. This may reflect a dimension from no formal to formal authorisation to teach mathematics. The factor was labelled *Basic competence*. The second factor includes items about how prepared the teacher is to teach most mathematics contents for grade 8. One indicator concerning didactic issues loading on this factor was: "how often teachers ask students to work with mathematics tasks in groups". Factor two was interpreted as *Mathematical didactic competence*. The third factor is dominated by items about how teachers instruct students and how they organise the mathematical work. The dimension is also indicated by items related to teachers´ ontological and didactical views of the academic subject mathematics. The dimension was labelled *Teacher intervention*.

***Discussion***

The dimensions of instructional modes obtained from the teacher data were *Basic competence*, *Mathematical didactic competence* and *Teacher intervention*. The first dimension could be viewed as "math teacher or not", the second as describing the quality of the math teachers` competence. The third dimension appeared in the analysis with parcelled items, while the first analysis with only single items did not identify any dimension concerning instructional indicators.

All together the exploratory factor analysis on the student- and teacher-data identified dimensions about instructional modes dealing with the school subject "mathematics" , both about how visible the subject was for the students and how confident teachers were with the subject. The analysis also showed in what way teachers got involved in students' work interaction.

An aim of the inquiry process was to discover and define latent variables and a measurement model that could provide the basis for the further confirmatory factor analysis. With EFA it has been possible to describe the proper number of common factors and to uncover which observed variables are reasonable indicators of the latent dimensions.

**Confirmatory factor analysis, CFA**

The results from the multilevel CFA will be presented next. First the intra-class correlation (ICC), with the class and individual level variances was calculated, se table 14. The results suggest that there are sizeable class effects, the ICCs ranging between 0.054-0.297.

[Take in Table 14 about here]

The resulting model for the within-class and the between-class structures for instructional modes are depicted in figure 1 and 2. The model shows reasonable to good “goodness-of-fit” indices: the comparative fit index CFI is 0.894 and the root mean square error of approximation RMSEA is 0.035. The former should be at least 0.95 but the latter is an acceptable value. Chi-square/df is about 5, which is too high, but “large N solutions are routinely rejected on the basis of chi-square…” (Brown, 2002, p. 81). Figure 1 presents the model for the within-class structure.

[Take in Fig 1 about here]

On the individual level, from the students´ point of view, there are three latent factors representing the dimensions in the instructional modes: *Math content*, *Teacher intervention* and *Peer interaction*. The model for the between-class structure is presented in Figure 2.

[Take in Fig 2 about here]

Between classes the same three instructional modes dimensions appear. But here the dimension Teacher intervention is identified both as *General teacher intervention* and as *Specific teacher intervention*.

### *Findings on the individual-level*

Table 15 shows the estimated factor loadings and the standard errors for the individual-, within-level, model.

[Take in Table 15 about here]

*Math content* is correlated 0.592 with *Teacher intervention* and 0.539 with *Peer interaction. Teacher intervention* correlates 0.672 with *Peer interaction.*

The first latent factor is indicated by items mirroring to what extent students experience math content during the math lessons. At first sight this seems strange, but this analysis highlights that this is a dimension that varies between students within classrooms.

The second factor is indicated by items mirroring to what extent students experience teachers to be intervening in the math education. The dimension includes items which capture instructional characteristics. In this dimension it is apparent if the teacher teaches and explains, but also if the teacher organises and arranges the working models.

The third factor was measured by observed variables concerning students´ opinion about to what extent there is a mix of autonomous work, which is a presumption for peer-interaction, and briefings from teachers. The item “explain our answers” is also indicating to what degree the peer effect is used in the math education.

All observed indicators in this within-classroom model are significant and the latent variable explains some part of the variation in the observed variables.

The value for S*tandardized Root Mean-Square Residual*, SRMR, was for the within part of the model 0.031 which suggests an appropriate model-fit on the student level.

### *Findings on the class-level*

Table 16 shows the estimated factor loadings and the standard errors for the class-level model.

[Take in Table 16 about here]

*General teacher intervention* is correlated 0.661 with *Peer interaction,* 0.430 with *Specific teacher intervention* and -0.651 with *Math content. Peer interaction* is correlated 0.223 with *Specific teacher intervention* and -0.283 with *Math content.* Finely is *Specific teacher intervention* correlated -0.379 with *Math content.*

A four-factor model will clarify the between-group analysis. The first and the third latent factors contain items about instructional characteristics, the first from students´ point of view and the third from teachers´ point of view. It is obvious that differences between classes due to instructional characteristics, from students´ point of view are indicated by similar items as were the differences on the individual-level. The differences explained by indicators from teachers´ questionnaire are referring to if students do relate what they learn to every day life or not and whether there is student-active work in groups or not. From the student questionnaire there are also indicators referring to math content, homework and tests. But these two latent factors are not entirely corresponding, because the model fit decreased when they were combined into one latent factor and the correlation between the factors is just 0.430. The first latent factor is labelled *General teacher intervention* since it brings up indicators referring to the fact that math problems are being explained, that the work is being organised and that content, homework and tests are realized. The third factor is labelled *Specific teacher intervention,* since it brings up indicators referring to one specific way of conducting the mathematics education. The dimension is characterised by the amount of student-active work in groups where the students decide by their own how to solve problems and where they do relate to everyday life what they are learning.

The second latent factor is labelled *Peer-interaction,* and items dealing with opportunities for interaction between peers in the instructional group are indicating the factor. What above all explains the variation between classes is to what extent teachers have long briefings in the math education. This was also an explanation between individuals, but not the strongest one. This dimension is from teachers' point of view indicated (negatively) by the item “if teachers ask student to solve complex problems”.

The dimension *Mathematical content* is represented in the fourth latent factor. It is indicated by items about students´ experiences of content topics in mathematic education. The item “how often do you practice addition, subtraction, multiplication and division” is dominating when compared to items concerning fractions and equations in explaining differences between classes. This was not the case on the individual-level. The three variables were equal in explaining the variation between individuals.

The value for SRMR was for the between part 0.106, which suggests there is still some room for improving the model-fit on the class-level.

### ***Discussion***

In this confirmatory factor analysis of the dimensions of instructional modes in the Swedish math education, it is established that *Teacher intervention, Peer-interaction* and *Math content* are together explaining an important part of the variation.

Teacher intervention is represented by two latent factors, *General teacher intervention* and *Specific teacher intervention.* In the prior there are observed variables referring to teachers explaining the math problems and to teachers governing the work in the education group. You could in this dimension expect to find math classrooms within a range from students “independent work” to high “teacher intervention”. At the former pole the students could autonomously do the planning, work individually at their own pace or together with whom they wanted to and could handle self-assessment. At the latter pole teachers were expected to explain math problems and govern most of the work in a direction of collaborative work, either in groups or all students together. So when there is a large share of teacher intervention, teachers are expected to engage the students to be active and they are managing the didactive planning and the instruction. In the latter latent factor, *Specific teacher intervention*, teachers' intervention is aimed to frame the work into groups where students autonomously do solve problems. Unlike the first latent factor, *Specific teacher intervention* does not only depict teachers to intervene in mathematics education, rather if they are intervening in a specific way, through causing students to work autonomously in groups with problem solving.

The two dimensions dealing with teachers intervention is not characterized by the ”traditional instructional mode”, neither by the “student-centred instructional mode”. Instead an underlying aspect of these two instructional modes has been obvious, namely in what way teachers are intervening in the math education. In the ”traditional instructional mode”, the teacher mainly explains procedures and gives direction (Hiebert et al., 2003; Porter, 1989; Silver & Smith, 1996; Stigler & Hiebert, 1997). In the “student-centred instruction” teachers are initiating interaction in learning which aims to develop the mathematical identity of the student (Ball & Bass, 2000; Boaler & Greeno, 2000; Cobb, Wood, & Yackel, 1993; Lampert, 2001; Yackel, Cobb, Wood, Wheatley, & Merckel, 1990). But while it is shown that descriptions of classrooms is not easily done in these teacher- and student-centred modes (Mok & Morris, 2001, Mok, 2003, Zuzovsky, 2008), the important underlying aspect “teacher intervention”, is preferred.

Items delineated in the analysis mirroring the interaction and active participation of the pupils are for example “if teachers ask students to work in groups”, “if students are asked to relate to every day life what they are learning in math” and ”if they are asked to decide on their own how to solve complex problems”. The dimension *General teacher intervention* explains differences between classes concerning if students´ do realise teachers to intervene in the math education or not and the dimension *Specific teacher intervention* explains differences between classes concerning if the work is organised in autonomous group-work or not. Between classes, but not between individuals, some of the variation in this dimension is explained by students’ experiences of math content, e.g. training fractions. With “intervening teachers” instead of “students’ learning by independent work”, a topic like fractions is perhaps more apparent for the students.

The dimension *Peer-interaction* is scaled between the ”traditional instructional mode” where the interaction between peers is not prominent and where teachers mainly explain procedures and give directions (Hiebert et al., 2003; Porter, 1989; Silver & Smith, 1996; Stigler & Hiebert, 1997) to the opposite pole of the scale with presumptions for peer-interaction. Common items for the individual- and the class-level indicating this dimension are “if students are listening to long briefings by the teacher”, “if they are asked to work autonomously with applied tasks” and “if they are asked to explain their answers”. From both student- and teacher-questionnaires the item “if teachers do ask students to decide on their own how to solve complex problems”, i.e. non traditional instruction mode, is an indicator important for the dimension *Peer-interaction*. Those latter observed variables do have negative factor-loadings when all other observed variables do have positive loadings.

The dimension *Mathematical content* is appropriate both on individual- and on class-level, indicated by the same items, although with different magnitudes. On the individual level there are three items indicating the dimension while on the class-level one item, “practice arithmetics” is dominating. This could probably indicate ability grouping. On an individual level however the item is indicating to what extent students are experiencing mathematics content in math education.

## Conclusion and Implications

The aim of this study was to develop a measurement model for instructional modes in the Swedish mathematics education. With the ambition to discuss the theoretical starting-points in the traditional division of the teacher- and student-centred modes, the dimensions found in this two-level structural equation modelling, have been examined aligned with theories crucial for mathematical learning for multilingual students. It is concluded that with the traditional division of instructional modes, there are underlying dimensions of teacher intervention and controlled peer-interaction, which may be hypothesized to be crucial for multilingual students’ math education. Those underlying dimensions of instructional modes in the Swedish mathematic education for 8th graders have been exposed in the measurement model evolved in this study which will also form the basis of my following structural equation modelling analysis.

One dimension, labelled *Teacher intervention* on the individual level and *General teacher intervention* on the group level, includes indicators dealing with presumptions for dialogues and interactions in the math classroom. It is represented by indicators which deal with the amount of teacher intervention that enables students to become active learners. This dimension is not fully comparable to the ”traditional instructional mode”, where the teacher mainly explains procedures and gives directions which students follow by carrying on doing individual work. However, there are similarities. As mentioned earlier, the ”student-centred instruction” is thought to be characterized by interaction between students when learning and the teacher is considered very important for initiating this interaction. Still the dimension emerged in this study is not labelled ”student-centred instruction”. This will not be done mainly in order to avoid the risk of misleading inferences. Students` “independent work” in the Swedish math education is however often regarded as ”student-centred instruction” although there is no teacher intervention in the education. A more relevant description of the dimension hence is *Teacher intervention*. A specific way of managing the interaction between students when learning is to organise the work in groups where students work autonomously with problem solving and where conceptual learning is in focus instead of procedural learning. Differences in instructional modes between classes are to some part explained by this dimension, *Specific teacher intervention*.

The dimension *Peer-interaction* does have similarities with the ”traditional instructional mode”. At one pole of this dimension you can find math education with teachers who give long briefings and where students work individually with applied tasks. But in the opposite pole there are opportunities for peer-interactions. Finally a third dimension *Math content* emerged. Considering evaluations of the Swedish math education, it is not astonishing to find that the math content is not naturally a part of the Swedish math education. Instead this dimension could explain if there is ability grouping in mathematics education or not.

The measurement model developed in this study opens up the possibility to acknowledge the importance of teachers' interventions and peer-interactions for multilingual students' math education. It is hypothesized that “students' independent work” without teacher intervention and controlled peer-interaction could promote “hidden differentiation”, where spontaneously formed ability groupings or individual work in the math education are made possible. This instructional mode does not support communicational processes hypothesized to be of particular significance for multilingual pupils' math learning. (Barwell & Clarkson, 2004; Hanushek & Wössmann, 2006; J. Moschkovich, 1999; J Moschkovich, 2002; Shayer & Adhami, 2007).

Hanushek and Wössmann (2006) showed that despite the fact that mathematics education is formally untracked in Sweden, test results with low averages and increasing dispersion, are similar to other countries' with an early differentiation. It should therefore be of interest to further examine student-centred modes of instruction in the Swedish mathematics education especially concerning the multilingual students. In a forthcoming inquiry this will be further investigated, and the mathematics topics as such will also be explored. Prior to further inquiry, however, the measurement model presented in this paper will be thoroughly challenged and elaborated.

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Table 1. List of Items and Scales, used in CFA

| Item (Student questionnaire) | | Scoring |
|---|---|---|
| BSBMHASM | How often do you practice addition, subtraction, multiplication and division? | 1(Every or almost every lesson)-4(never) |
| BSBMHWFD | How often do you train fractions? | 1(Every or almost every lesson)-4(never) |
| BSBMHEFR | How often do you train equations? | 1(Every or almost every lesson)-4(never) |
| BSBMHWSG | How often do you work together in small groups? | 1(Every or almost every lesson)-4(never) |
| BSBMHMDL | How often do you relate to your every day life? | 1(Every or almost every lesson)-4(never) |
| BSBMHEXP | How often do you explain your answers? | 1(Every or almost every lesson)-4(never) |
| BSBMHSCP | How often do you decide on your own how to solve complex problems? | 1(Every or almost every lesson)-4(never) |
| BSBMHROH | How often do you penetrate your homework in the class? | 1(Every or almost every lesson)-4(never) |
| BSBMHLSP | How often do you listen to long briefings by the teacher? | 1(Every or almost every lesson)-4(never) |
| BSBMHWPO | How often do you work autonomously with applied tasks? | 1(Every or almost every lesson)-4(never) |
| BSBMHBHC | How often do you make your homework during the lessons? | 1(Every or almost every lesson)-4(never) |
| BSBMHHQT | How often do you have tests and probations? | 1(Every or almost every lesson)-4(never) |
| **Item (Teacher questionnaire)** | | **Scoring** |
| BTBMASDL | In math lessons, how often do you ask students to relate what they learn to every day life? | 1(Every or almost every lesson)-4(never) |
| BTBMASSG | In math lessons, how often do you ask students to work together in small groups? | 1(Every or almost every lesson)-4(never) |
| BTBMASCP | In math lessons, how often do you ask students to decide how to solve complex problems on their own? | 1(Every or almost every lesson)-4(never) |

Table 2. List of Items used in EFA, from Student questionnaire

| Observed variable | Concerning |
|---|---|
| BSBMHWFD | How often do you train fractions? |
| BSBMHMDL | How often do you relate to your every day life? |
| BSBMHSCP | How often do your decide by your self how to solve complex problems? |
| BSBMHWPO | How often do you work autonomous with applied tasks? |
| BSBMHCAL | How often do you use calculator? |
| BSBMHASM | How often do you practice addition, subtraction, multiplication and division? |
| BSBMHGCT | How often do you train charts, diagrams and graph? |
| BSBMHEFR | How often do you train equations? |
| BSBMHWSG | How often do you work together in small groups? |
| BSBMHEXP | How often do you explain your answers? |
| BSBMHROH | How often do you penetrate your homework in the class? |
| BSBMHLSP | How often do you listen to long briefings by the teacher? |
| BSBMHBHC | How often do you make your homework during the lessons? |
| BSBMHHQT | How often do you have tests and probations? |

Table 3. List of Items used in EFA, from Teacher questionnaire

| Observed variable | Concerning |
|---|---|
| BTBMRE01 | Relate to your education and your experiences of both subject content and instruction in mathematic, how prepared are you to teach within … decimal- and fractional numbers? |
| BTBMRE02 | ... integers? |
| BTBMRE03 | … numeric, algebraic and geometric pattern and sequence of numbers? |
| BTBMRE04 | ... equations and equation systems? |
| BTBMRE05 | … representations of functions? |
| BTBMRE06 | … characteristic features of graphs? |
| BTBMRE07 | … estimations of lengths, perimeters, spaces, volumes…? |
| BTBMRE08 | … numeration by measures in applied tasks? |
| BTBMRE09 | …the size of irregular or composite areas? |
| BTBMRE10 | … accuracy when measuring? |
| BTBMRE11 | … Pythagorean theorem? |
| BTBMRE12 | … congruent figures? |
| BTBMRE13 | … right-angled coordinate plane? |
| BTBMRE14 | … transpositions, mirroring, rotation and enlarging? |
| BTBMRE15 | … error source when collecting and compilation of data? |
| BTBMRE16 | …methods for collecting data? |
| BTBMRE17 | … characteristics of data sets? |
| BTBMRE18 | …simple probability…? |
| BTBGOHGT | How many hours per week do you usually spend on correcting and evaluating tests, besides the regular school-day? |
| BTBGOHPL | How many hours per week do you usually spend on planning lessons, besides the regular school-day? |
| BTBGOHAT | How many hours per week do you usually spend on administrative duties, besides the regular school-day? |
| BTBGOTDC | How often do you cooperate with other teachers by discussing how to instruct for a special topic? |
| BTBGOTMP | How often do you cooperate with other teachers to prepare school supplies? |
| BTBGOTVT | How often do you visit other teachers to observe how they instruct? |
| BTBGOTAT | How often do other teachers visit your classroom? |
| BTBMPDMT | During the two last years, have you participated in further training of the subject matter in mathematics? |
| BTBMPDMP | During the two last years, have you participated in further training of pedagogic/methodology in mathematics? |
| BTBMPDMC | During the two last years, have you participated in further training in curricula for mathematics? |
| BTBMPDIT | During the two last years, have you participated in further training for integration of IT in mathematics? |
| BTBGPDCT | During the two last years, have you participated in further training in developing students' critical thinking or ability of problem solving? |
| BTBMPDMA | During the two last years, have you participated in further training in evaluation of students´ cognitions in mathematics? |
| BTBMADMR | To which extent do you agree; …many representations ought to be used… |
| BTBMADSA | To which extent do you agree;…mathematics ought to be learnt as a set of procedures and roles… |
| BTBMADHY | To which extent do you agree;…to solve problems is to hypothesise, making ratings, tests and… |
| BTBMADME | To which extent do you agree;…to learn mathematics is mainly to memorise. |
| BTBMADDW | To which extent do you agree;…there are different ways of solving mostly problems in mathematics. |
| BTBMADFD | To which extent do you agree;… new discoveries in mathematics are being made. |
| BTBMADRW | To which extent do you agree;…to model problems close to reality is foundational in mathematics. |
| BTBMPTRH | Which part of a lesson in mathematics do students spend on working with homework**?** |
| BTBMPTLS | Which part of a lesson in mathematics do students spend on listening to long briefings? |
| BTBMPTYG | Which part of a lesson in mathematics do students spend on working with tasks under your guidance? |

| | |
|---|---|
| BTBMPTOO | Which part of a lesson in mathematics do students spend on working autonomous? |
| BTBMPTRT | Which part of a lesson in mathematics do students spend on listening when you are repeating and further explaining? |
| BTBMPTTQ | Which part of a lesson in mathematics do students spend on taking tests and probations? |
| BTBMPTCM | Which part of a lesson in mathematics do students spend on other activities then mathematics? |
| BTBMASPC | How often do you ask students to practise addition, subtraction, multiplication and division without calculator? |
| BTBMASWF | How often do you ask students to work with fractions and decimal numbers? |
| BTBMASWS | How often do you ask students to work with problems which methods are not obvious? |
| BTBMASOD | How often do you ask students to interpret charts, diagrams and graph? |
| BTBMASRR | How often do you ask students to depict equations and functions…? |
| BTBMASSG | How often do you ask students to work together in small groups? |
| BTBMASDL | How often do you ask students to relate what they learn to every day life? |
| BTBMASEA | How often do you ask students to explain their answers? |
| BTBMASCP | How often do you ask students to decide how to solve complex problems on their own? |
| BTBGLT01 | To which degree do you think that students with different abilities limit your instruction? |
| BTBGLT02 | To which degree do you think that students with different backgrounds are limiting your instruction? |
| BTBGLT03 | To which degree do you think that students with special needs are limiting your instruction? |
| BTBGLT04 | To which degree do you think that uninterested students are limiting your instruction? |
| BTBGLT05 | To which degree do you think that bad spirit among students is limiting your instruction? |
| BTBGLT06 | To which degree do you think that interfering students are limiting your instruction? |
| BTBMHDAM | How often do you control if homework is done? |
| BTBMHDAF | Howe often do you assess homework and give feed-back? |
| BTBMHDAC | How often do your students assess homework by themselves? |
| BTBMHDAD | How often do you use homework as a starting-point for discussion? |
| BTBMHDAG | How often do you use homework as foundation for grading? |

Table 4. Pooled-Within-analysis. Inter-factor correlations. (Quartimin rotated loadings)

| | Factor 1 | Factor 2 | Factor 3 |
|---|---|---|---|
| Factor 1 | 1.000 | | |
| Factor 2 | 0.388 | 1.000 | |
| Factor 3 | -0.426 | -0.326 | 1.000 |

Table 5. Pooled-Within-analysis. Pattern coefficient matrix. (Quartimin rotated loadings)

| | Concerning | Factor 1 *Mathematic content* | Factor 2 *Teacher intervention* | Factor 3 *Peer-interaction* |
|---|---|---|---|---|
| BSBMHWFD | How often do you train fractions? | **0.820** | -0.069 | -0.002 |
| BSBMHMDL | How often relate to your every day life? | 0.147 | **0.339** | -0.201 |
| BSBMHSCP | How often do you decide on your own how to solve complex problems? | 0.036 | 0.010 | **-0.580** |
| BSBMHWPO | How often do you work autonomously with applied tasks? | 0.108 | -0.204 | **-0.387** |
| BSBMHCAL | How often do you use a calculator? | 0.089 | -0.026 | -0.142 |
| BSBMHASM | How often do you practice addition, subtraction, multiplication and division? | **0.535** | -0.069 | -0.079 |
| BSBMHGCT | How often do you train charts, diagrams and graph? | **0.668** | 0.139 | 0.031 |
| BSBMHEFR | How often do you train equations? | **0.518** | 0.143 | -0.033 |
| BSBMHWSG | How often do you work together in small groups? | 0.078 | **0.436** | 0.021 |
| BSBMHEXP | How often do you explain your answers? | -0.045 | 0.049 | **-0.588** |
| BSBMHROH | How often do you penetrate your homework in the class? | -0.029 | **0.374** | -0.230 |
| BSBMHLSP | How often do you listen to long briefings by the teacher? | 0.027 | 0.062 | **-0.358** |
| BSBMHBHC | How often do you make your homework during the lessons? | 0.014 | **0.364** | -0.025 |
| BSBMHHQT | How often do you have tests and probations? | 0.180 | **0.341** | -0.014 |

*Notes.* Values above +/- 0.30 are indicated in bold.

Table 6. Between-group-analysis. Inter-factor correlations. (Quartimin rotated loadings)

| | Factor 1B | Factor 2B | Factor 3B |
|---|---|---|---|
| Factor 1 | 1.000 | | |
| Factor 2 | 0.253 | 1.000 | |
| Factor 3 | 0.133 | 0.073 | 1.000 |

Table 7. Between-group-analysis. Pattern coefficient matrix. (Quartimin rotated loadings)

| Item | Concerning | Factor 1B *Mathematic content* | Factor 2B *Teacher intervention* | Factor 3B *Peer-interaction* |
|---|---|---|---|---|
| BSBMHWFD | How often do you practice fractions? | **0.375** | **0.349** | 0.181 |
| BSBMHMDL | How often relate to your every day life? | 0.140 | **0.829** | -0.006 |
| BSBMHSCP | How often do you decide on your own how to solve complex problems? | 0.000 | 0.052 | **0.414** |
| BSBMHWPO | How often do you work autonomously with applied tasks? | -0.048 | **-0.324** | **0.767** |
| BSBMHCAL | How often do you use calculator? | **-0.483** | -0.090 | **0.303** |
| BSBMHASM | How often do you practice addition, subtraction, multiplication and division? | **1.385** | -0.013 | 0.000 |
| BSBMHGCT | How often do you practice charts, diagrams and graph? | 0.014 | **0.385** | 0.168 |
| BSBMHEFR | How often do you practice equations? | 0.261 | 0.078 | **0.554** |
| BSBMHWSG | How often do you work together in small groups? | -0.011 | **0.514** | **-0.389** |
| BSBMHEXP | How often do you explain your answers? | -0.032 | **0.458** | **0.630** |
| BSBMHROH | How often do you penetrate your home-work in the class? | 0.033 | **0.699** | 0.035 |
| BSBMHLSP | How often do you listen to long briefings by the teacher? | 0.106 | **0.531** | **0.489** |
| BSBMHBHC | How often do you make your homework during the lessons? | -0.130 | **0.367** | -0.032 |
| BSBMHHQT | How often do you have tests and probations? | 0.180 | **0.800** | -0.121 |

*Notes.* Values above +/- 0.30 are indicated in bold.

Table 8. Parceled observed variables, student-data

| Parcel | Item | Concerning |
|---|---|---|
| CON | BSBMHWFD, | How often do you train fractions? |
| | BSBMHASM | How often do you practice addition, subtraction, multiplication and division? |
| DEC | BSBMHMDL | How often relate to your every day life? |
| | BSBMHROH | How often do you penetrate your homework in the class? |
| | BSBMHHQT | How often do you have tests and probations? |
| | BSBMHWSG | How often do you work together in small groups? |

Single-items used in the modelling (factor-loadings>0.20 in the “single-item-approach”); BSBMHSCP, BSBMHWPO, BSBMHCAL, BSBMHGCT, BSBMHEFR, BSBMHEXP, BSBMHLSP, BSBMHBHC

Table 9. Inter-factor correlations for EFA, teacher-data. "Single-item-model". (Quartimin rotated loadings)

| | Factor 1 | Factor 2 | Factor 3 | Factor 4 |
|---|---|---|---|---|
| Factor 1 | 1.000 | | | |
| Factor 2 | 0.467 | 1.000 | | |
| Factor 3 | 0.197 | 0.135 | 1.000 | |
| Factor 4 | 0.087 | 0.131 | 0.035 | 1.000 |

Table 10. Factor loading for EFA, teacher-data. "Single-item-model".

| Item | Factor 1 | Factor 2 | Factor 3 | Factor 4 |
|---|---|---|---|---|
| BSBMRE01 | **0.965** | -0.043 | 0.024 | 0.005 |
| BSBMRE02 | **0.938** | -0.001 | 0.038 | 0.018 |
| BSBMRE03 | **0.323** | **0.498** | -0.032 | 0.034 |
| BSBMRE04 | 0.081 | **0.797** | -0.041 | -0.021 |
| BSBMRE05 | 0.130 | **0.716** | 0.015 | -0.103 |
| BSBMRE06 | 0.001 | **0.850** | 0.045 | 0.010 |
| BSBMRE07 | **0.541** | 0.294 | -0.066 | -0.003 |
| BSBMRE08 | **0.635** | 0.271 | 0.015 | -0.050 |
| BSBMRE09 | **0.385** | **0.459** | 0.085 | 0.068 |
| BSBMRE10 | 0.147 | **0.665** | -0.034 | 0.017 |
| BSBMRE11 | 0.203 | **0.620** | -0.011 | -0.057 |
| BSBMRE12 | 0.187 | **0.610** | 0.061 | 0.108 |
| BSBMRE13 | -0.009 | **0.842** | 0.032 | 0.017 |
| BSBMRE14 | -0.058 | **0.763** | -0.063 | 0.054 |
| BSBMRE15 | -0.173 | **0.728** | -0.047 | -0.009 |
| BSBMRE16 | -0.151 | **0.705** | 0.144 | 0.100 |
| BSBMRE17 | -0.089 | **0.670** | -0.018 | -0.050 |
| BSBMRE18 | -0.027 | **0.567** | -0.005 | 0.021 |
| BSBGOHGT | -0.009 | -0.008 | -0.038 | -0.109 |
| BSBGOHPL | 0.042 | 0.104 | -0.087 | -0.063 |
| BSBGOHAT | 0.111 | -0.098 | -0.199 | -0.141 |
| BSBGOTDC | 0.097 | -0.129 | -0.092 | 0.007 |
| BSBGOTPM | 0.007 | -0.100 | -0.129 | -0.075 |
| BSBGOTVT | 0.039 | -0.180 | 0.066 | -0.047 |
| BSBGOTAT | -0.073 | -0.068 | 0.045 | 0.015 |
| BSBMPDMT | -0.016 | 0.099 | **0.685** | -0.016 |
| BSBMPDMP | 0.006 | 0.004 | **0.634** | 0.002 |
| BSBMPDMC | 0.018 | 0.017 | **0.611** | 0.016 |
| BSBMPDIT | 0.023 | -0.074 | **0.455** | 0.019 |
| BSBGPDCT | 0.053 | -0.133 | **0.627** | 0.042 |
| BSBMPDMA | 0.007 | 0.017 | **0.663** | -0.101 |
| BSBMADMR | 0.071 | 0.002 | 0.163 | 0.028 |
| BSBMADSA | -0.169 | 0.107 | -0.039 | -0.226 |
| BSBMADHY | 0.108 | 0.005 | -0.045 | 0.089 |
| BSBMADME | -0.073 | 0.047 | 0.023 | -0.245 |
| BSBMADDW | 0.067 | 0.110 | 0.137 | 0.035 |
| BSBMADFD | -0.050 | -0.044 | -0.076 | -0.061 |
| BSBMADRW | -0.031 | -0.029 | -0.009 | 0.100 |
| BSBMPTRH | -0.114 | 0.010 | -0.095 | -0.095 |
| BSBMPTLS | -0.029 | -0.075 | -0.096 | -0.041 |
| BSBMPTYG | 0.000 | 0.131 | 0.029 | 0.128 |
| BSBMPTOO | -0.026 | -0.066 | 0.062 | -0.120 |
| BSBMPTRT | 0.005 | -0.061 | -0.017 | -0.062 |

| | | | | |
|---|---|---|---|---|
| BSBMPTTQ | 0.118 | -0.106 | -0.135 | -0.106 |
| BSBMPTCM | 0.214 | -0.117 | 0.055 | 0.269 |
| BSBMASPC | 0.198 | -0.026 | -0.104 | -0.016 |
| BSBMASWF | 0.056 | -0.036 | 0.130 | 0.061 |
| BSBMASWS | -0.033 | 0.120 | 0.057 | 0.062 |
| BSBMASID | -0.069 | 0.166 | 0.139 | -0.001 |
| BSBMASRR | -0.058 | 0.151 | 0.108 | 0.002 |
| BSBMASSG | -0.098 | 0.241 | 0.105 | 0.057 |
| BSBMASDL | 0.074 | 0.052 | 0.271 | 0.228 |
| BSBMASEA | 0.091 | 0.058 | 0.053 | 0.185 |
| BSBMASCP | -0.001 | 0.006 | 0.126 | 0.230 |
| BSBGLT01 | 0.093 | -0.080 | 0.012 | **0.488** |
| BSBGLT02 | 0.076 | 0.024 | -0.152 | **0.440** |
| BSBGLT03 | 0.146 | -0.044 | -0.106 | **0.461** |
| BSBGLT04 | -0.034 | 0.034 | 0.055 | **0.775** |
| BSBGLT05 | -0.052 | 0.079 | -0.060 | **0.795** |
| BSBGLT06 | 0.012 | -0.059 | 0.018 | **0.786** |
| BSBMHDAM | 0.035 | 0.019 | 0.131 | 0.150 |
| BSBMHDAF | 0.156 | -0.176 | 0.088 | 0.153 |
| BSBMHDAC | -0.151 | 0.121 | 0.142 | -0.187 |
| BSBMHDAD | 0.186 | 0.025 | 0.198 | 0.128 |
| BSBMHDAG | 0.130 | 0.009 | 0.108 | -0.137 |

*Notes.* Values above +/- 0.30 are indicated in bold.

Table 11. Items used in the modelling. Single-items and parcelled items. Teacher-data.

| Parcel | Item | Concerning |
|---|---|---|
| COM1 | BTBMRE03, 07,08, 09 | Basic subject competence for mathematic teachers |
| COM2 | BTBMRE04, 05, 06, 10, 11, 12, 13, 14, 15, 16, 17,18 | General subject competence for mathematic teachers |
| TRA | BTBMPDMT, BTBMPDMP, BTBMPDMC, BTBMPDIT, BTBGPDCT, BTBMPDMA | Further training and capacity building for mathematics teachers |
| ONT | BTBMADSA, BTBMADME | Ontological and didactical views of the academic subject mathematics |
| LIM | BTBGLT01, 02, 03, 04, 05, 06 | In what way teachers experience that different student behaviours and characters are limiting for their teaching |

Table 12. Inter-factor correlations for EFA, teacher-data. *"Item-parcelling-approach*". (Quartimin rotated loadings)

| | Factor 1 | Factor 2 | Factor 3 |
|---|---|---|---|
| Factor 1 | 1.000 | | |
| Factor 2 | 0.570 | 1.000 | |
| Factor 3 | 0.162 | 0.172 | 1.000 |

Table 13. Factor loading for EFA, teacher-data. *"Item-parceling-approach"*. (Quartimin rotated loadings)

| Item | Concerning | Factor 1 *Basic competence* | Factor 2 *Mathematical didactive competence* | Factor 3 *Teacher intervention* |
|---|---|---|---|---|
| BTBMRE01 | Prepared to educate within decimal- and fractional numbers? | **0.990** | -0.025 | 0.022 |
| BTBMRE02 | Prepared to educate within integers? | **0.894** | 0.069 | -0.001 |
| BTBMPTCM | Students spend time on other activities than mathematics | 0.242 | -0.094 | 0.132 |
| BTBMASSG | Work together in small groups | -0.186 | 0.270 | **0.357** |
| BTBMASDL | Relating what they learn to every day life | 0.078 | 0.008 | **0.580** |
| BTBMASCP | Students decide how to solve complex problems | -0.006 | -0.044 | **0.568** |
| COM1 | Basic subject competence | **0.347** | **0.660** | -0.024 |
| COM2 | General subject competence | -0.069 | **0.870** | 0.020 |
| TRA | Further training | 0.148 | 0.043 | 0.174 |
| ONT | View of the academic subject | -0.131 | 0.090 | **-0.342** |
| LIM | Limited by different behaviours | 0.015 | 0.099 | 0.265 |

*Notes.* Values above +/- 0.30 are indicated in bold.

Table 14. Intra class correlation (ICC) for student data items

| Variable | ICC | Variable | ICC |
|---|---|---|---|
| BSBMHASM | 0.098 | BSBMHROH | 0.297 |
| BSBMHWSG | 0.280 | BSBMHBHC | 0.230 |
| SBMHSCP | 0.054 | BSBMHEFR | 0.113 |
| BSBMHWPO | 0.054 | BSBMHEXP | 0.082 |
| BSBMHWFD | 0.067 | BSBMHLSP | 0.202 |
| BSBMHMDL | 0.090 | BSBMHHQT | 0.074 |

Table 15. Individual level estimates (standardized model results) and standard errors. Within-level.

| Item | Latent Factor 1: *Math content* | Latent Factor 2: *Teacher intervention* | Latent Factor 3: *Peer interaction* |
|---|---|---|---|
| BSBMHASM<br>Practice arithmetic | 0,514 (0.019) | | |
| BSBMHWFD<br>Train fractions | 0.560 (0.017) | | |
| BSBMHEFR<br>Train equations | 0.419 (0.019) | | |
| BSBMHWSG<br>Work in groups | | 0.302 (0.022) | |
| BSBMHMDL<br>Relay to weekday | | 0.506 (0.021) | |
| BSBMHROH<br>Penetrate homework | | 0.400 (0.022) | |
| BSBMHWPO<br>Work autonomous | | -0.232 (0.051) | |
| BSBMHBHC<br>Homework during lessons | | 0.258 (0.023) | |
| BSBMHHQT<br>Tests and probations | | 0.255 (0.019) | |
| BSBMHEXP<br>Explain answers | | | 0.535 (0.021) |
| BSBMHSCP<br>Decide autonomously how to solve problems | | | 0.557 (0.020) |
| BSBMHLSP<br>Listen to briefings | | | 0.320 (0.019) |
| BSBMHWPO<br>Work autonomously with applied tasks | | | 0.484 (0.053) |

*Notes.* ns= non significant (Two-tailed Est./S.E.<2.0, P-value>0.05)

Table 16. Class level estimates

| Item | Latent Factor 1B: *General teacher intervention* | Latent Factor 2B: *Peer interaction* | Latent Factor 3B: *Specific teacher intervention* | Latent Factor 4B: *Math content* |
|---|---|---|---|---|
| BSBMHASM Practice arithmetics | 0.321 ns (0.171) | | | 0.329 (0.135) |
| BSBMHWFD Train fractions | 0.179 (0.068) | | | 0.148 (0.049) |
| BSBMHWSG Work in groups | 0.270 (0.062) | | | |
| BSBMHMDL Relate to every day life | 0.218 (0.045) | | | |
| BSBMHSCP Decide on their own | 0.001 ns (0.055) | | | |
| BSBMHROH Penetrate homework | 0.386 (0.055) | | | |
| BSBMHWPO Work autonomously with applied tasks | -0.213 (0.052) | | | |
| BSBMHBHC Homework during lessons | 0.157 (0.038) | | | |
| BSBMHHQT Tests and probations | 0.114 (0.025) | | | |
| BSBMHEFR Train equations | | 0.179 (0.055) | | 0.117 (0.036) |
| BSBMHEXP Explain answers | | 0.189 (0.048) | | |
| BSBMHSCP Decide on your own how to solve complex problems | | 0.025 ns (0.063) | | |
| BSBMHLSP Listen to briefings | | 0.344 (0.054) | | |
| BSBMHWPO Work autonomously | | 0.173 (0.058) | | |
| BTBMASCP Decide on their own how to solve complex problems | | -0.194 (0.102) | | |
| BTBMASDL Ask students to relate to every day life | | | 0.349 (0.085) | |
| BTBMASSG Ask students to work in groups | | | 0.378 (0.119) | |
| BTBMASCP Ask students to decide on their own how to solve complex problems | | | 0.419 (0.126) | |

*Notes.* ns= non significant (Two-tailed Est./S.E.<2.0, P-value>0.05)

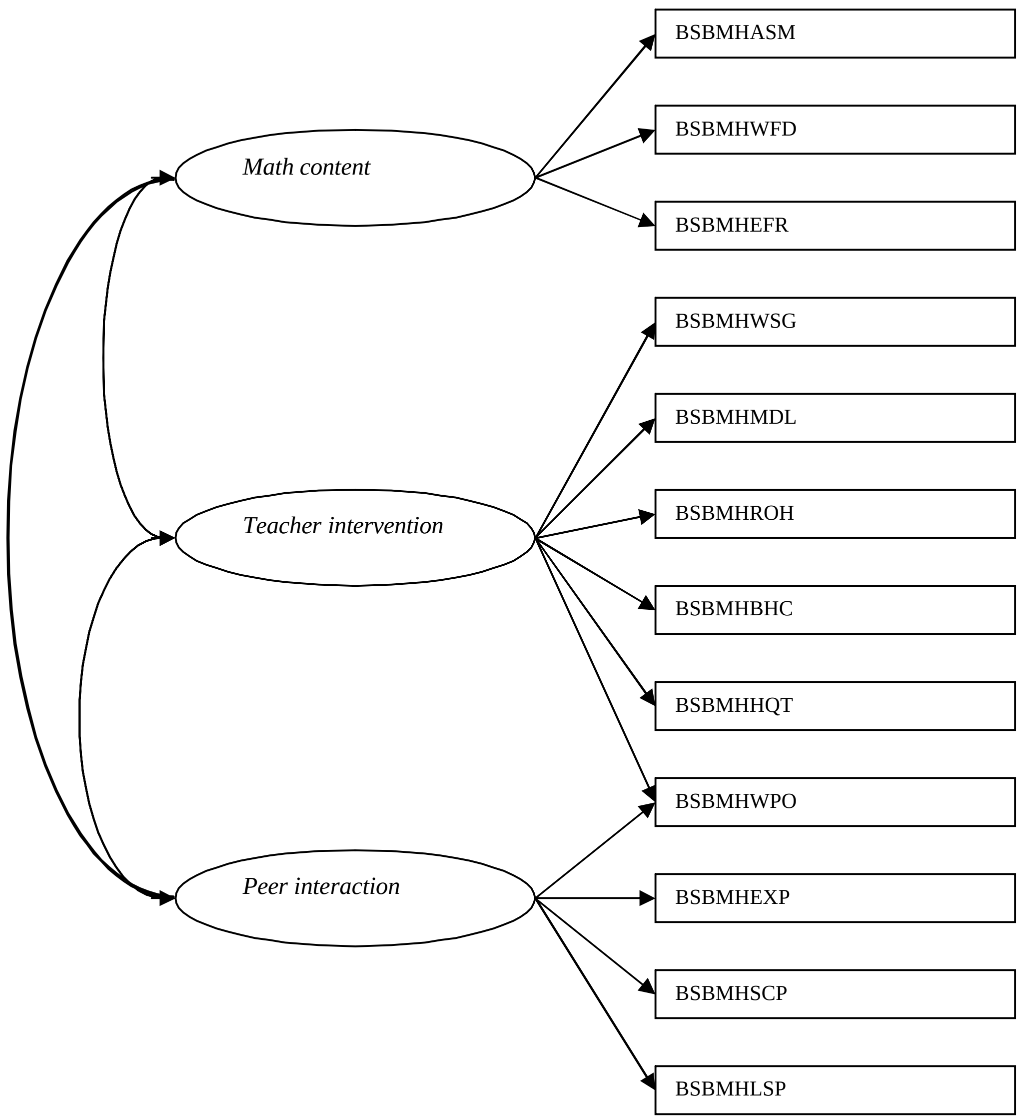


Figure 1. Multilevel Model: Measurement model for Instructional modes, Within-level

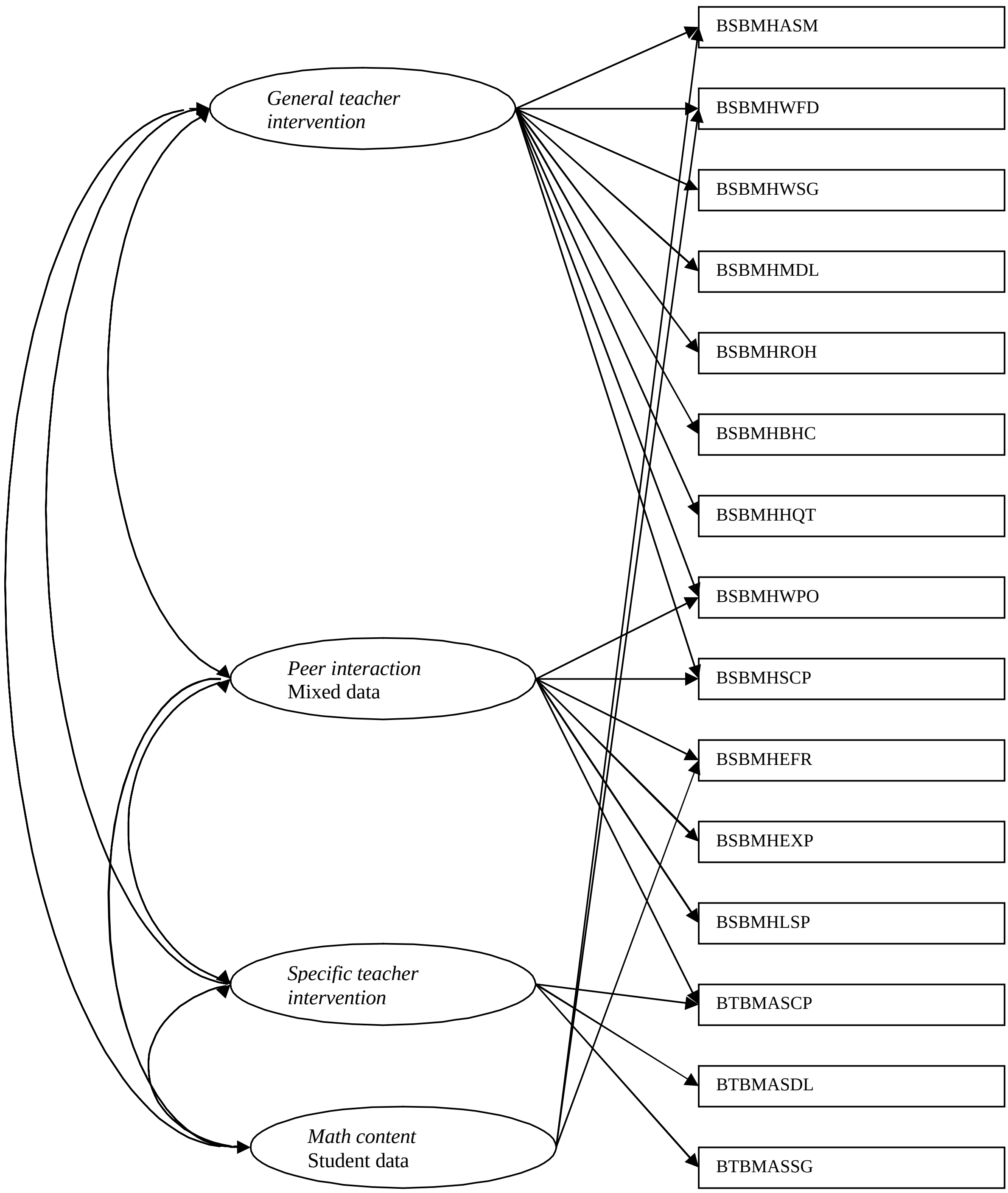


Figure 2. Multilevel Model: Measurement model for Instructional modes, Between-level